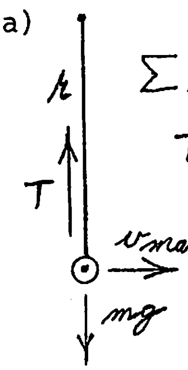
John B. Johnston, The Faraday Center, 103 Creamery Rd., Livingston Manor, NY 12758
(845) 439 - 4706

Tension in a pendulum - As you observe its swing, you notice that the bob's velocity keeps changing. Further study reveals that the acceleration involved is constantly changing, also. In other words, things get complicated quickly, something that fascinated Galileo in the 1580's. Of course, the root cause of all the changing is the changing force (tension) in the string. It is due to two different forces combined, the force due to gravity ($W_t = mg$) and the centripetal effect (mv^2/r) and depends on the bob's position in the cycle. An analysis of the component force of mg along the string during the swing provides a simple, but rewarding, exercise in the use of vector diagrams.

The purpose of this demonstration is to show students that the force (tension) in the string really does vary and where the maximum and minimum amounts of force are located. Some teachers like to begin the discussion with the question, if the string were to break, at what point would this most likely occur?

Presentation: Using diagrams (a) and (b) with the two most important positions of the pendulum, the bottom of its swing (a) and the top of its swing (b), we can logically come up with a good estimate of the force (tension) in the string (spring) if we know the mass (1 kg = 9.8 nt) of the bob and the amplitude (release point angles like 30°, etc.).

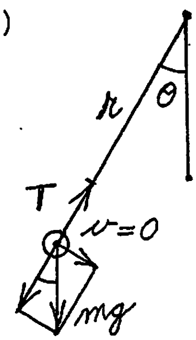
diagram: (a)



$$\Sigma F_r = T - mg = m\frac{v^2}{r}$$

$$T_{max} = m\left(g + \frac{v^2}{r}\right)$$

(b)



$$\Sigma F_r = T - mg\cos\theta = m\frac{v^2}{r}$$

$$T = m\left(g\cos\theta + \frac{v^2}{r}\right)$$

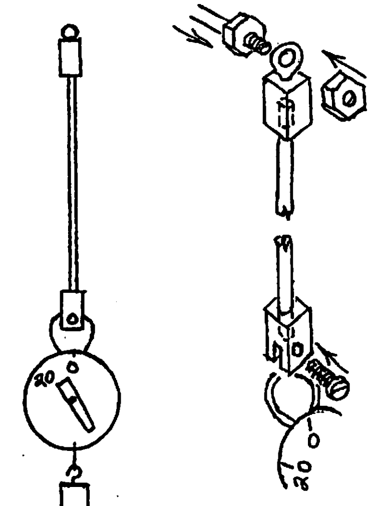
$$\text{at } \theta = \theta_{max} \quad v = 0$$

$$T_{min} = mg\cos\theta_{max}$$

In (a) the gravitational force component and centripetal effect are at their maxima. In (b) the component force of mg along the string is reduced by $\cos\theta$ and the centripetal effect drops to zero. Of course, in every position between (a) and (b), the tension will be changing between the maximum of (a) and the minimum of (b).

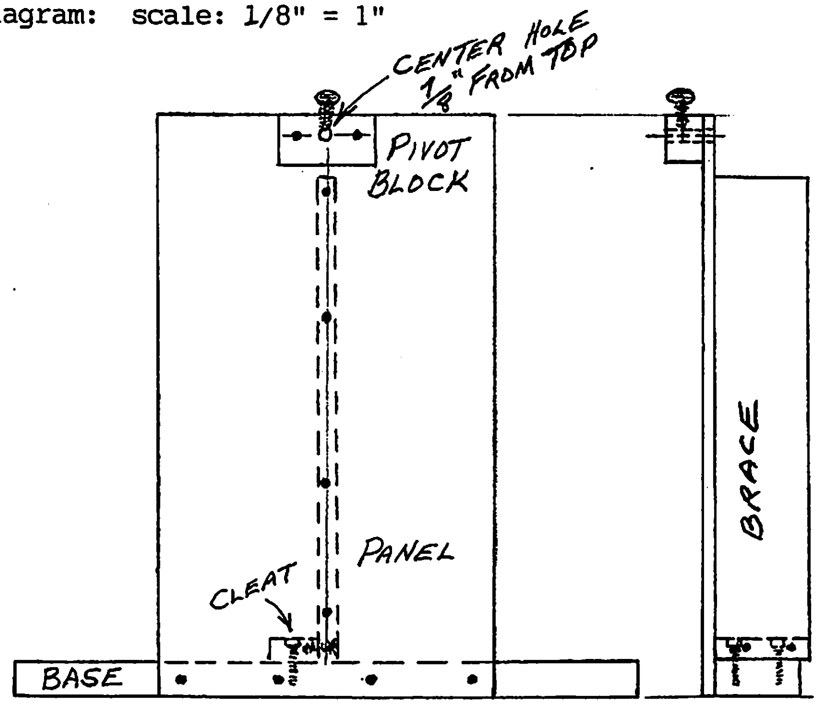
Originally, the pendulum was set up in the simplest way possible, using a string, but the scale twisted so badly on the string during the swing due to the changing tension that the students couldn't read it. (Maybe you will have better luck.) Instead, I substituted a specially made dowel rod to eliminate the twisting. The present apparatus swings well enough for satisfactory readings by the students. (Maybe you will have an even better idea.)

Construction: The dimensions of the pendulum are arbitrary; you may wish to change them. For a spring scale, the S-W or Ohaus, 20 nt, 8" dia. dial-type works well. Choose a 15½" long, 3/8" dia. dowel for the "string". Cut out two 5/4" square x 1 3/4" long pine blocks for the end pieces. Drill a 3/8" dia. hole in each block, 3/4" deep for the dowel (and glue). In the "bottom" block, cut a 1/8" slot, 3/4" deep for the scale's loop; on its side, drill a 5/32" dia. hole to accept a 1"long #10-24 RH stove bolt. force the bolt to self-thread into the pine. In the "top" block, select a screweye with a 3/8"+ inside dia. to accept a 3/8" dia. steel rod with two 3/8"-16 nuts made from a 3/8", 4" long machine bolt. (Cut off the head of the bolt.) Drill a small hole in the end of the top block so the screweye fits tightly but not so tight that you split the block.



Based on your resources, you must decide how you want to mount the pendulum for display. Remember, a swinging, 1 kg, hooked mass will want to "sway" the pivot point which needs to be kept motionless (rigid) for best results. A plan of my "rigid" stand in included; if you have a better plan, use it.

diagram: scale: 1/8" = 1"



Base: a 26" piece of "2 x 4"

Panel: a 14" x 24" piece of ½" plywood

Brace: a 4" x 20" piece of 3/4" pine

Cleat: a 2" x 4" block of 1" pine

Pivot block: a 2" x 4" block, 1½" thick with a 3/8" dia. hole for the pivot rod and a 15/64" hole for the ¼"-20, 3/4" thumb screw which will self-thread nicely

Clamp the ends of the base to the edge of a table top.

Conclusion: From this demo, your students should have learned that the physics of the simple pendulum is not so simple because everything is always changing. One of the first to recognize this complexity was Newton when he began to analyze the pendulum as Galileo had done. He immediately realized that the mathematics of his day was not powerful enough to solve the changing tension in a simpler, more direct way. This and other physics problems he addressed created a need for a more powerful math; thus, he pursued a new method and developed the calculus. This exercise is a nice way to show the transition from algebra and trigonometry to calculus, a definite step up as a means to solve problems.