Oscillations & Simple Harmonic Motion

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Oscillations

- Any motion that repeats itself in equal intervals of time is called periodic or harmonic motion.
- The period, \( T \), is the time required for one complete oscillation or cycle.
- The frequency, \( f \), is the number of oscillations per unit time.
- The unit of frequency is Hertz (Hz)
  - 1 Hz = 1 cycle/s
- The frequency is the reciprocal of the period
  \[ f = \frac{1}{T} \]
Simple Harmonic Motion

- When a particle is under the effect of a linear restoring force, the resulting motion is a special type of oscillatory motion called simple harmonic motion.

- For example, a block attached to a spring on a frictionless surface moves in simple harmonic motion.
Mass-Spring System

\[
F = ma \\
-kx = m\frac{d^2x}{dt^2} \\
\frac{d^2x}{dt^2} + \frac{k}{m}x = 0 \\
x(t) = A \cos(\omega t + \phi)
\]
Mass-Spring System Constants

\[ x(t) = A \cos(\omega t + \phi) \]
\[ \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \]
\[ \frac{d^2x}{dt^2} = -\omega^2 A \cos(\omega t + \phi) \]
\[
\frac{d^2x}{dt^2} + \frac{k}{m}x = 0
\]

\[-\omega^2 A \cos(\omega t + \phi) + \frac{k}{m}A \cos(\omega t + \phi) = 0\]
\[ \omega = \sqrt{\frac{k}{m}} \]

The constants \(A\) and \(\phi\) are arbitrary and depend on how the motion starts.
Angular Frequency

- Suppose we increase $t$ by $\frac{2\pi}{\omega}$

$$x = A \cos \left[ \omega \left( t + \frac{2\pi}{\omega} \right) + \phi \right]$$

$$= A \cos (\omega t + 2\pi + \phi)$$

$$= A \cos (\omega t + \phi)$$

- The function repeats itself after a time $\frac{2\pi}{\omega}$, so the period of the motion is

$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}$$

- The frequency of the motion is

$$f = \frac{1}{T} = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

- The angular frequency of the motion is

$$\omega = 2\pi f = \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$
Amplitude and Phase

\[ x(t) = A \cos(\omega t + \phi) \]

- \( A \) is called the amplitude of the motion
- \((\omega t + \phi)\) is called the phase of the motion
- \(\phi\) is called the phase constant
Displacement, Velocity & Acceleration in SHM

\[ x = A \cos (\omega t + \phi) \]
\[ x_{\text{max}} = A \]

\[ v = \frac{dx}{dt} = -\omega A \sin (\omega t + \phi) \]
\[ v_{\text{max}} = \omega A = \sqrt{\frac{k}{m}} A \]

\[ a = \frac{dv}{dt} = -\omega^2 A \cos (\omega t + \phi) \]
\[ a_{\text{max}} = \omega^2 A = \frac{k}{m} A \]
Displacement, Velocity & Acceleration in SHM (cont’d)
Example

A block with a mass of 0.680 kg is fastened to a spring with a spring constant of 65.0 N/m. The block is pulled a distance $x = 0.110$ m from its equilibrium position at $x = 0$ on a frictionless surface and released from rest at $t = 0$. Find (a) the angular frequency, (b) the frequency, (c) the period, (d) the amplitude, (e) the maximum speed, (f) the maximum acceleration, (g) the phase constant, and (h) the displacement function for the motion.
Example Solution

(a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{65.0 \text{ N/m}}{0.680 \text{ kg}}} = 9.78 \text{ rad/s}$

(b) $f = \frac{\omega}{2\pi} = \frac{9.78 \text{ rad/s}}{2\pi \text{ rad}} = 1.56 \text{ Hz}$

(c) $T = \frac{1}{f} = \frac{1}{1.56 \text{ Hz}} = 0.641 \text{ s}$

(d) $x_{max} = A = 0.110 \text{ m}$

(e) $v_{max} = \omega A = (9.78 \text{ rad/s}) (0.110 \text{ m}) = 1.08 \text{ m/s}$

(f) $a_{max} = \omega^2 A = (9.78 \text{ rad/s})^2 (0.110 \text{ m}) = 10.5 \text{ m/s}^2$

(g) $\phi = 0$

(h) $x(t) = A \cos (\omega t + \phi)$

$x(t) = (0.110 \text{ m}) \cos [(9.78 \text{ rad/s}) t]$
Homework Set 22 - Due Fri. Nov. 5

- Read Sections 12.1-12.2
- Answer Questions 12.3 & 12.4
- Do Problems 12.3, 12.4, 12.6 & 12.14