The Charge to Mass Ratio of the Electron

1 Introduction

In this exercise we will measure the charge to mass ratio of the electron by accelerating an electron beam through a known potential and measuring the radius of curvature of the beam in a known magnetic field. This technique is similar to the one used by J. J. Thomson in 1897 to make the first measurement of this ratio.

The apparatus that we will use in this lab is shown in Figure 1. The main components are a round glass bulb that holds helium atoms at low pressure and two coils to produce a magnetic field in the region of the bulb. A filament inside the bulb is supplied with electric current to make it glow and eject electrons. These electrons are accelerated toward an anode that is held at an electric potential V above the filament, which also serves as the cathode. The electrons pass through a slit in the anode and form an electron beam. As the electrons collide with the helium atoms they give them energy and promote them into excited states. These excited atoms return to lower energy levels very rapidly and emit photons. As a result we can visibly "see" the path of the electron beam in the helium.

Figure 1: The e/m apparatus. (Picture taken from http://store.pasco.com.)

The two coils on either side of the bulb have the same number of windings of wire and the same radius. Also, the coils are separated by a distance equal to the radius of the coils. This configuration which is called a Helmholtz coil, named after the German Scientist Hermann von Helmholtz (try the site: http://scienceworld.wolfram.com/biography/Helmholtz.html), produces a uniform magnetic field in the region of the bulb that points perpendicular to the planes of the coils. By increas-
ing the current though the coils we can increase the strength of the magnetic field and thus bend the path of the electron beam into a full circle.

2 Derivation of the Expression for $\frac{q}{m}$

When a charged particle moves perpendicular to a magnetic field it experiences a force that is perpendicular to both its velocity and the direction of the magnetic field. (In other words, the force is normal to the plane containing the velocity and the magnetic field.) The magnitude of the force is given by

$$F_m = qvB$$  \hspace{1cm} (1)

where $q$ is the charge of the particle in Coulombs (C), $v$ is the speed of the particle in m/s, and $B$ is the strength of the magnetic field in tesla (T). Because the force is normal to the velocity of the particle, the direction of motion is changed, but not the speed and the particle undergoes uniform circular motion.

The acceleration of a particle in uniform circular motion is

$$a = \frac{v^2}{r}$$ \hspace{1cm} (2)

where $r$ is the radius of the circular path. From Newton’s second law of motion, the net force on the particle is

$$F_n = ma = m\frac{v^2}{r}$$ \hspace{1cm} (3)

where $m$ is the mass of the particle. If the magnetic force is the only force acting on the particle (or if any other force, like the gravitational force on an electron, is negligibly small) then it is also the net force on the particle and we can write

$$F_m = m\frac{v^2}{r}.$$ \hspace{1cm} (4)

Combining Eqs. (1) and (4) we have

$$qvB = m\frac{v^2}{r}$$ \hspace{1cm} (5)

and

$$\frac{q}{m} = \frac{v}{rB}.$$ \hspace{1cm} (6)

In this experiment, electrons are produced at a hot filament and accelerated through a measured potential difference. The kinetic energy gain is equal to the work done on the charged electron, so we can write

$$\frac{1}{2}mv^2 = qV$$ \hspace{1cm} (7)

where $V$ is the electric potential difference. Solving for the speed of the electron, we have

$$v = \sqrt{\frac{2qV}{m}}.$$ \hspace{1cm} (8)
Substituting Eq. (8) into Eq. (6), we have

$$\frac{q}{m} = \frac{2V}{r^2B^2}. \tag{9}$$

The magnetic field is produced by a pair of circular coils carrying an electric current. In this arrangement (known as Helmholtz coils) the separation of the coils is equal to the radius of a coil and the magnetic field is almost uniform near the midpoint between the coils. The magnetic field, in tesla, is given by

$$B = 7.80 \times 10^{-4}I \tag{10}$$

where $I$ is the current in the coils in amps. Substituting Eq. (10) into Eq. (9), we get the final expression for the charge to mass ratio of the electron in C/kg

$$\frac{q}{m} = 3.29 \times 10^6 \frac{V}{r^2I^2} \tag{11}$$

where $V$ is the measured potential difference in volts (V), $r$ is the measured radius of the circular path in meters (m), and $I$ is the measured current in amps (A).

### 3 Measuring $q/m$

1. Turn on the current in the filament. The filament voltage should be already set at 6.3 V. Do not adjust it.

2. Set the potential difference to 200 V. With no current in the Helmholtz coils a horizontal beam of electrons is produced which should be visible in the tube. The intensity of the emitted light is low, so the background light needs to be reduced as much as possible.

3. Turn on the current in the Helmholtz coil and increase it until the electron beam makes a complete circle. Make sure that the plane of the circle is parallel to the planes of the coils. If this is not so, rotate the bulb so that it becomes parallel. Record the values of the coil current, $I$, and the accelerating potential, $V$, in the table below.

4. Look through the tube and line up the left edge of the electron beam with its reflection in the mirrored scale behind the bulb. Record the position of the left edge as $P_1$ in the table below.

5. Look through the tube and line up the right edge of the electron beam with its reflection in the mirrored scale behind the bulb. Record the position of the left edge as $P_2$ in the table below.

6. Increase the accelerating potential to 250 V and repeat steps 3-5.

7. Calculate the radius of the circular path of the electron beam, $r$, from your measurements of $P_1$ and $P_2$ for each trial and record the results in the data table.

8. Use Eq. (11) to calculate $q/m$ for each trial and record the results in the data table. Also, show the calculations in Section 4.
9. Calculate the average q/m and record the result in the space provided in Section 4.

10. Compare your average value for q/m with the accepted value for the charge to mass ratio of the electron of $1.76 \times 10^{11}$ C/kg by calculating the % difference given by

$$\% \text{ difference} = \frac{\text{your value} - \text{accepted value}}{\text{accepted value}} \times 100\%.$$  \hspace{1cm} (12)

Record the result in the space provided in Section 4.

4 Data and Calculations

<table>
<thead>
<tr>
<th>V (V)</th>
<th>I (A)</th>
<th>P₁ (m)</th>
<th>P₂ (m)</th>
<th>r (m)</th>
<th>q/m (C/kg)</th>
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$$\left( \frac{q}{m} \right)_{\text{Ave.}} = \quad \text{C/kg}$$

% difference = \quad \% \\

Calculations: