Some Formulas

\[ W = \mathbf{F} \cdot \Delta \mathbf{r} \]
\[ W = \int_{x_c}^{x_f} F(x) \, dx \]
\[ W_{\text{net}} = \Delta K \]
\[ K = \frac{1}{2} m \mathbf{v}^2 \]
\[ U_g = m g z \]
\[ U_{sp} = \frac{1}{2} k x^2 \]
\[ F_{sp} = -kx \]
\[ \mathbf{p} = m \mathbf{v} \quad I = \int_{t_i}^{t_f} \mathbf{F}(t) \, dt = \Delta \mathbf{p} \]
\[ v_{f,1} = \left( \frac{m_1 - m_2}{m_1 + m_2} \right) v_{i,1} + \left( \frac{2m_2}{m_1 + m_2} \right) v_{i,2} \]
\[ v_{f,2} = \left( \frac{2m_1}{m_1 + m_2} \right) v_{i,1} + \left( \frac{m_2 - m_1}{m_1 + m_2} \right) v_{i,2} \]
\[ X_{\text{CM}} = \frac{\sum_{i} m_i x_i}{M} \]

\[ \mathbf{p}_{\text{tot}} = M \mathbf{v}_{\text{CM}} \]
\[ F_{\text{ext}} = \frac{d\mathbf{p}_{\text{tot}}}{dt} = M \mathbf{a}_{\text{CM}} \]
\[ \Delta t = \gamma \Delta t_p \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \]
\[ L = \frac{1}{\gamma} L_p \]
\[ p = \gamma m v \]
\[ E = \gamma m c^2 \]
\[ E' = E - mc^2 \]
Part I Solve the problems posed in the space provided. Show your work.

1. A block of mass $M = 25.0 \text{ kg}$ is pushed a distance $D = 10.00 \text{ m}$ across a rough, horizontal floor by a force of magnitude $F = 35.0 \text{ N}$ directed at an angle $25.0^\circ$ downward from horizontal, as shown. At the end of that distance, the block is moving at a speed of $3.00 \text{ m/s}$.

   ![Diagram showing a block being pushed with a force directed downward at an angle]

   a) How much work was done by the pushing force during the motion?

   $$W_{\text{push}} = F \cdot \Delta \vec{r} = |F| |\Delta \vec{r}| \cos 25^\circ$$

   $$W = 35.0 \text{ N} \cdot 10.0 \text{ m} \cdot 0.906 = 317 \text{ J}$$

   b) How much work was done by the normal force on the block during the pushing?

   $$F_N \cdot \Delta \vec{r} = 0$$

   c) What is the kinetic energy of the block at the end of the trip?

   $$K = \frac{1}{2} M v^2 = 0.5 \times 25.0 \text{ kg} \cdot (3.00 \text{ m/s})^2$$

   $$K_f = 112.5 \text{ J}$$

   d) How large is the frictional force on the block?

   As a consequence of friction, $W_{\text{push}} - K_f = 317 \text{ J} - 112.5 \text{ J} = 204.5 \text{ J}$ has been converted to internal energy.

   $$W_{\text{push}} - K_f = f_k D$$

   $$f_k = \frac{204.5 \text{ J}}{10.0 \text{ m}} = 20.4 \text{ N}$$
2. A ball of mass \( m = 1.5 \text{ kg} \) is dropped from rest. After falling a distance \( h \) the ball lands on a spring of force constant \( k = 1470.0 \text{ N/m} \), giving it a maximum compression \( d = 3.0 \text{ cm} \).

\[
\begin{align*}
U_{s, f} &= \frac{1}{2} k x_f^2 = \frac{1}{2} (1470.0 \text{ N/m}) (0.03 \text{ m})^2 \\
&= 0.662 \text{ J}
\end{align*}
\]

a) How much potential energy was stored in the spring at maximum compression?

b) Find the distance \( h \).

3. A drive-by shooting occurs in a space colony. The only witness to the crime reports that the perpetrator passed by in a space ship that was 100 meters long at a speed of 0.25 \( c \) and that the driver was an ugly space alien that hiccuped every 5.0 seconds. When the space police visit the homes of the likely suspects...

3. A drive-by shooting occurs in a space colony. The only witness to the crime reports that the perpetrator passed by in a space ship that was 100 meters long at a speed of 0.25 \( c \) and that the driver was an ugly space alien that hiccuped every 5.0 seconds. When the space police visit the homes of the likely suspects...

\[ V = 0.25 c \Rightarrow \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1.03 \]

a) What rate of hiccups, in number of hiccups per minute, should the space police look for in a suspect?

\[
\Delta t = 6 \Delta t_p \quad (\Delta t_p \text{ is the time between hiccups according to the police and the crook})
\]

\[
\Delta t_p = \frac{\Delta t}{6} = 4.85 \text{ s}
\]

\[
12.4 \text{ hiccups per minute}
\]

b) What length of ship should the space police look for in the garages?

\[
\Delta L = \frac{\Delta t}{\Delta t_p} \Rightarrow \Delta L_p = \gamma \Delta L = 1.03 \times 100 \text{ m}
\]

\[
\Delta L_p = 103 \text{ m}
\]
4. A set of three train cars \((M_A = M, M_B = 2.50 \text{ M}, M_C = 1.50 \text{ M})\), where \(M = 5000 \text{ kg}\), is being assembled by colliding the cars on a frictionless, straight track. A spring-loaded latch system is designed to couple them together. Initially, car A moves with speed 5.00 m/s to the right and catches up with car B (whose initial speed of 2.00 m/s was also to the right). After sticking together, cars A+B collide with car C, but the latch is broken, and the cars (A+B vs C) instead bounce in a perfectly elastic collision.

a) What is the total initial momentum of the system A B C?
\[
p_{\text{tot}} = p_{A} + p_{B} = M \left( 5 \frac{\text{kg}}{\text{s}} \cdot \frac{5}{3} \text{ m/s} + 2.50 \cdot (2.00 \text{ m/s}) \right)^{\uparrow}
\]
\[
p_{\text{tot}} = 5000 \text{ kg} \cdot 10 \text{ m/s} = 5.00 \times 10^{4} \text{ kg m/s} \quad \text{to right}
\]

b) What is the total initial kinetic energy of the system?
\[
k_{i} = k_{A} + k_{B} \quad \text{since} \quad k_{C} = 0 \quad \text{initially.}
\]
\[
k_{i} = \frac{1}{2} M \left[ (5.00 \text{ m/s})^{2} + 2.5 \cdot (2.00 \text{ m/s})^{2} \right]
\]
\[
k_{i} = 2500 \text{ kg} \cdot \left[ 35 \text{ m}^{2}/\text{s}^{2} \right] = 8.75 \times 10^{4} \text{ J}
\]

c) What is the speed of cars A+B after they have collided and stuck, but before striking car C?
\[
v_{A+B} = \frac{p_{A} + p_{B}}{M_{A} + M_{B}} = \frac{p_{\text{tot}, i}}{M_{A+B}} = \frac{5.00 \times 10^{4} \text{ kg m/s}}{3.5 \times 5.00 \times 10^{3} \text{ kg}}
\]
\[
v_{A+B} = \frac{10}{3.5} \text{ m/s} = \frac{2.86 \text{ m/s}}{}
\]

d) What is the velocity of car C after being struck?
\[
v_{C} = \frac{2 \left( m_{1} \right)}{m_{1} + m_{2}} v_{1 i} + \left( \frac{m_{2} - m_{1}}{m_{1} + m_{2}} \right) v_{2 i}
\]
\[
v_{C} = 2 \left( \frac{3.5 \text{ kg}}{3.5 + 1.5} \right) \cdot 2.86 \text{ m/s} = 4.00 \text{ m/s} \quad \text{to right}
\]

e) What is the final velocity of cars A+B after striking car C?
\[
p_{f, \text{tot}} = p_{i, \text{tot}} \quad \text{for the system.}
\]
\[
M_{A+B} v_{A+B} + M_{C} v_{C} = 5.00 \times 10^{4} \text{ kg m/s} \quad \text{from part (a)}
\]
\[
v_{A+B, f} = \frac{5.00 \times 10^{4} \text{ kg m/s} - 1.5 \cdot 5.000 \text{ kg} \cdot 4.00 \text{ m/s}}{3.5 \times 5.00 \times 10^{3} \text{ kg}} = \frac{1.14 \text{ m/s}}{}
\]

f) How much kinetic energy was lost in all the collisions? Explain where this energy went and when.
\[
k_{f} = k_{A+B} + k_{C} = \frac{1}{2} M \left\{ 3.5 \left( \frac{1.14 \text{ m/s}}{3.5 \text{ m/s}} \right)^{2} + 1.5 \left( 4.00 \text{ m/s} \right)^{2} \right\} = 7.14 \times 10^{4} \text{ J}
\]
\[
k_{f} - k_{i} = (8.75 \times 10^{4} - 7.14) \times 10^{4} \text{ J} = 1.61 \times 10^{4} \text{ J} = 16.1 \text{kJ}
\]

This energy was lost in the inelastic collision, which is now spring potential and heat.
Part II

5. An object moving along the x-axis is acted upon by a single force $F$ that varies with position as shown.

\[ W = \int F \, dx \]
\[ = \frac{1}{2} (2m)(8N) + (4m)(8N) \]
\[ = 40J \]

\[ \begin{array}{c}
\text{F (N)} \\
\hline
8 \\
4 \\
-4 \\
-8 \\
\text{x (m)}
\end{array} \]

a) How much work is done by this force as the object moves from $x = 2$ m to $x = 8$ m?
A. -10 J  B. +10 J  C. +30 J  D. -30 J  E. +40 J

b) The speed of the object at $x = 2$ m will be _____ the speed at $x = 8$.
A. less than  B. equal to  C. greater than

\[ k_f = k_i + 40J \]

6. The figure shows the magnitudes and directions of two vectors. Find the dot product of these two vectors.

\[ \vec{F} \cdot \vec{\Delta r} = |\vec{F}| |\Delta r| \cos 20^\circ \]
\[ = (32.8 N)(17.3 m)(0.940) = 533 J \]

7. After colliding, two bodies move away with different velocities. Which of the following statements must be true? (circle one)
A. Mechanical Energy is conserved.
B. Linear Momentum is conserved.
C. The collision was perfectly inelastic.
D. Both bodies must have been moving before the collision.
E. The force of interaction in the collision was non-conservative.
8. A light beam passes by an observer at rest on the earth and also it passes an observer moving at speed 0.5 c relative to the earth in the same direction as the light beam. The speed of the light beam as measured by the moving observer is ...
   A. greater than c
   B. equal to c
   C. less than c

9. Inside a space ship, a space alien flying at a relativistic speed near the rear of the ship sees a space rodent straight ahead which it soon catches without changing its velocity. Consider the \( \Delta t \) between the moment the alien sees the rodent and when it catches the rodent, and consider the distance the alien flew from the moment it saw the rodent to when it caught the rodent. In which reference frame or frames are the proper time between these events and the proper length of this distance? (circle one)
   A. The proper time and proper length are both in the hawk’s reference frame.
   B. The proper time is in the hawk’s reference frame and the proper length is in the rodent’s.
   C. The proper time is in the rodent’s reference frame and the proper length is in the hawk’s.
   D. The proper time and proper length are both in the rodent’s reference frame.