Physics 111 Homework Solutions Week #10

Wednesday, March 4, 2009
Chapter 24
Questions
24.2  In a photoelectric experiment the intensity refers to the number of photons incident on the metal surface per second. Let a beam of green light with intensity I be incident on the surface and note that it produces a photocurrent. If we switch to blue light of the same intensity I, the energy of the individual photons is greater and thus the maximum kinetic energy of the ejected photoelectrons is greater. The intensity is a constant, which is the total energy of the photons in the beam of light. The total energy is a product of the energy of each photon multiplied by the total number of photons. If the energy of each individual photon increases then the number of photons in the beam decreases since we want the total energy of the beam (which is proportional to the intensity) to remain constant. Since the number of photons decreases the number of photoelectrons decreases and the photocurrent decreases.

24.4  If for a particular wavelength of green light we find a stopping potential of –1.5V, this allows us to determine the work function \( \phi \), or the minimum energy needed to eject a photoelectron. Now, if we switch to a blue light the wavelength is shorter than that of green light and the maximum kinetic energy of the ejected photoelectrons will thus be greater using blue light as opposed to green light.

Multiple-Choice
24.2  B
24.3  C
24.4  D
24.6  C

Problems
24.8 Photoelectric effect in cesium
We are given the work function for cesium is \( \phi = 2.9 \text{ eV} = 4.64 \times 10^{-19} \text{ J} \).

a. The maximum wavelength corresponds to the minimum frequency.

Therefore, \( KE_{\text{min}} = hf_{\text{min}} - \phi = 0 \) which gives

\[
f_{\text{min}} = \frac{\phi}{h} = \frac{4.64 \times 10^{-19} \text{ J}}{6.63 \times 10^{-34} \text{ Js}} = 7.0 \times 10^{14} \text{ s}^{-1}
\]

From

\[
c = f_{\text{min}} \lambda_{\text{max}} \rightarrow \lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{3 \times 10^8 \text{ m/s}}{7.0 \times 10^{14} \text{ s}^{-1}} = 4.28 \times 10^{-7} \text{ m} = 428 \text{ nm}
\]

b. If 400nm photons are used, their energy is given by
\[ E = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{m/s}}{400 \times 10^{-9} \text{m}} = 4.97 \times 10^{-19} \text{J} \times \frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}} = 3.11 \text{eV} \, .
\]
Therefore the maximum kinetic energy is given as
\[ KE_{\text{max}} = hf - \phi = 3.11 \text{eV} - 2.9 \text{eV} = 0.21 \text{eV} \times \frac{1.6 \times 10^{-19} \text{J}}{1 \text{eV}} = 3.33 \times 10^{-20} \text{J} \, .
\]

24.9 The maximum KE is given by the product of the stopping potential (0.82V) and the electron’s charge (e⁻). Thus the maximum \( KE = eV_{\text{stop}} = 0.82 \text{eV} \, . \) This is equal to the energy of the photons incident minus the work function (the minimum energy needed to eject a photoelectron). In symbols,
\[
KE_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi \\
\rightarrow 0.82 \text{eV} = \left( \frac{6.63 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{m/s}}{400 \times 10^{-9} \text{m}} \right) \times \frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}} - \phi \rightarrow \phi = 2.29 \text{eV}
\]

24.10 A photoelectric effect experiment
a. The maximum kinetic energy is given by
\[
KE_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi \\
KE_{\text{max}} = \left( \frac{6.63 \times 10^{-34} \text{Js} \times 3 \times 10^8 \text{m/s}}{410 \times 10^{-9} \text{m}} \right) \times \frac{1 \text{eV}}{1.6 \times 10^{-19} \text{J}} - 2.28 \text{eV} \\
KE_{\text{max}} = 3.03 \text{eV} - 2.28 \text{eV} = 0.75 \text{eV}
\]
In order to calculate the speed, we use the expression for the relativistic kinetic energy. We have
\[ KE = 0.75eV = (\gamma - 1)mc^2 = (\gamma - 1)0.511MeV \]
\[ \gamma = 1 + \frac{0.75eV}{0.511 \times 10^6 eV} = 1 + 0000015 = 1.000015 \]
and the speed is therefore \[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v^2 = 2.94 \times 10^{-6}c^2 \rightarrow v = 0.0017c \]

b. Based on the speed calculated in part a, the electron is not relativistic.

c. The minimum frequency corresponds to an electron ejected with a kinetic energy equal to zero. Therefore, \( KE_{\text{min}} = hf_{\text{min}} - \phi = 0 \) which gives
\[ f_{\text{min}} = \frac{\phi}{h} = \frac{2.28eV \times 1.6 \times 10^{-19} J}{6.63 \times 10^{-34} Js} = 5.5 \times 10^{14} \text{ s}^{-1} . \]

d. The minimum frequency corresponds to the maximum wavelength. Therefore we have
\[ c = f_{\text{min}}\lambda_{\text{max}} \rightarrow \lambda_{\text{max}} = \frac{c}{f_{\text{min}}} = \frac{3 \times 10^8 \frac{m}{s}}{5.5 \times 10^{14} \text{ s}^{-1}} = 5.45 \times 10^{-7} m = 545nm . \]

e. For 700nm photons we have the maximum kinetic energy given as
\[ KE_{\text{max}} = hf - \phi = \frac{hc}{\lambda} - \phi \]
\[ KE_{\text{max}} = \left( \frac{6.63 \times 10^{-34} Js \times 3 \times 10^8 \frac{m}{s}}{700 \times 10^{-9} m} \right) \times \frac{1eV}{1.6 \times 10^{-19} J} - 2.28eV \]
\[ KE_{\text{max}} = 1.78eV - 2.28eV = -0.50eV \]
Or, we have that no photoelectrons are produced. In addition we know that the maximum wavelength for photo production is 545nm, and we are well beyond this, so no photocurrent would be produced.

Friday, March 6, 2009
Exam #3
Questions
- None

Multiple-Choice
- None

Problems
- None
The atomic number $Z$ is the number of protons in the nucleus. It distinguishes the different types of atoms. $N$ is the number of neutrons in the atom. If we sum the number of neutrons ($N$) and the number of protons ($Z$) we get the mass of the nucleus (and the atom if we assume the mass of the electrons are negligible).

$\alpha$ decay is characterized by the following reaction: $^{A}_{Z}X \rightarrow ^{4}_{2}He + ^{A-4}_{Z-2}Y$ therefore the mass number of the nucleus decreases by 4 and the atomic number decreases by 2.

$\beta$ decay (for a high speed electron) is characterized by the following reaction: $^{A}_{Z}X \rightarrow ^{0}_{-1}e + ^{A}_{Z+1}Y$ therefore the mass number of the nucleus is unaffected and the atomic number increases by 1.

$\beta$ decay (for a high speed positron) is characterized by the following reaction: $^{A}_{Z}X \rightarrow ^{0}_{+1}e + ^{A}_{Z-1}Y$ therefore the mass number of the nucleus is unaffected and the atomic number decreases by 1.

$\gamma$ decay is characterized by the following reaction: $^{A}_{Z}X \rightarrow ^{0}_{0} \gamma + ^{A}_{Z}X$ therefore the mass number of the nucleus and the atomic number remain unchanged.

The three requirements for stability are as follows. 1) The number of neutrons in the nucleus. As more protons are packed into the nucleus those nuclides with significantly more neutrons than protons will tend to produce a stable nucleus. The extra neutrons tend to shield the individual protons from one another. 2) The binding energy of the nucleus. 3) The nuclear energy levels (like those of the electron) should be closed shells. That is the most stable nuclei have equal numbers of protons and neutrons (these numbers are called the magic numbers).

Multiple-Choice
26.1  D
26.2  A
26.4  A

Problems
26.2  A neutron star
   a) To calculate the mass of the neutron star we need to know the density of nuclear material and the volume of the star. We model the star as being spherical so it has a volume given by $\frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1 \times 10^4 \text{ m})^3 = 4.19 \times 10^{12} \text{ m}^3$. Given that the nuclear density is $2 \times 10^{17} \text{ kg/m}^3$ (assuming the nuclear radius is $1.2 \times 10^{-15} \text{ m}$) we have the mass $M = \rho V = 8.4 \times 10^{29} \text{ kg}$. Since the mass of the sun is $2 \times 10^{30}$kg this gives the mass of the neutron star as 0.41 solar masses. As an aside, this is almost $\frac{1}{2}$ the mass of the sun packed in to a sphere of radius 10 km $\sim$ 6 miles!!
b) Assuming that the mass of a nucleon (either a proton or neutron) is $1.67 \times 10^{-27}$ kg, this gives the number of nucleons as $(8.4 \times 10^{29} \text{ kg}) / (1.67 \times 10^{-27} \text{ kg/nucleon}) = 5.03 \times 10^{56} \text{ particles.}$

26.3 The density is given by $\rho = \frac{M}{V}$, $V = 4\pi r^3 = \frac{M}{\rho}$ and therefore the radius is given as $r = \left( \frac{M}{4\pi \rho} \right)^{\frac{1}{3}} = \left( \frac{2 \times 10^{30} \text{ kg}}{4\pi (2 \times 10^{17} \text{ kg/m}^3)} \right)^{\frac{1}{3}} = 11,675 \text{ m} = 11.7 \text{ km.}$

26.4 The nuclear binding energy is given through:

$$NBE = Z m_p c^2 + N m_N c^2 - m_{atom} c^2;$$

where $m_p = 1.00727 u$ and $m_n = 1.00867 u$.

For radium-226:

$$NBE = \left[ 88(1.00727 u) + 138(1.00867 u) - 225.97709 u \right] c^2 \times \frac{931.5 \text{ MeV}}{uc^2} = 1731.8 \text{ MeV.}$$

The $\frac{NBE}{\text{ nucleon}} = \frac{1731.8 \text{ MeV}}{226} = 7.66 \text{ MeV/nucleon.}$

For radium-228:

$$NBE = \left[ 88(1.00727 u) + 140(1.00867 u) - 227.98275 u \right] c^2 \times \frac{931.5 \text{ MeV}}{uc^2} = 1742.7 \text{ MeV.}$$

The $\frac{NBE}{\text{ nucleon}} = \frac{1742.7 \text{ MeV}}{228} = 7.64 \text{ MeV/nucleon.}$

For thorium-232:

$$NBE = \left[ 90(1.00727 u) + 142(1.00867 u) - 231.98864 u \right] c^2 \times \frac{931.5 \text{ MeV}}{uc^2} = 1766.9 \text{ MeV.}$$

The $\frac{NBE}{\text{ nucleon}} = \frac{1766.9 \text{ MeV}}{232} = 7.62 \text{ MeV/nucleon.}$

26.6 For the $\beta$-decay reaction of $^{24}\text{Na}$,

$$Q = \left( M_{^{24}\text{Na}} - M_{^{24}\text{Mg}} - M_{e^-} \right) c^2$$

$$\therefore Q = \left( 23.98492 - 23.97845 - 5.49 \times 10^{-4} \right) uc^2 \times \frac{931.5 \text{ MeV}}{uc^2} = 5.52 \text{ MeV.}$$

26.7 For alpha decay: $Q = \left( M_{\text{parent}} - M_{\text{daughter}} - M_{\text{He}} \right) c^2$. If the parent is at rest when it decays, then from conservation of momentum, the daughter gets a recoil velocity in the direction opposite direction to the velocity of the alpha particle.

Conservation of momentum gives:

$$0 = -m_{\text{daughter}} v_{\text{daughter}} + m_\alpha v_\alpha \rightarrow v_{\text{daughter}} = \frac{m_\alpha}{m_{\text{daughter}}} v_\alpha.$$ 

Next we apply conservation of energy and we find: 

$$m_{\text{parent}} c^2 = \frac{1}{2} m_{\text{daughter}} v_{\text{daughter}}^2 + \frac{1}{2} m_{\text{He}} v_{\text{He}}^2 + m_{\text{daughter}} c^2 + m_{\text{He}} c^2.$$
Then we can replace the velocity of the recoiling daughter atom in terms of the velocity of the alpha particle, which can be measured. We have

\[ m_{\text{parent}}c^2 = \frac{1}{2} m_{\text{daughter}} \left( \frac{m_\alpha}{m_{\text{daughter}}} v_\alpha \right)^2 + \frac{1}{2} m_{\text{He}}v_{\text{He}}^2 + m_{\text{daughter}}c^2 + m_{\text{He}}c^2. \]

Bringing all of the rest of the energy terms to one side and combining the terms involving the velocity of the alpha particle we find that

\[ (m_{\text{parent}}c^2 - m_{\text{daughter}}c^2 - m_{\text{He}}c^2) = Q = \left[ \frac{1}{2} \frac{m_\alpha^2}{m_{\text{daughter}}} + \frac{1}{2} m_\alpha \right] v_\alpha. \]

Factoring out the mass of the alpha particle on the left hand side we have

\[ Q = \frac{1}{2} m_\alpha v_\alpha^2 \left[ 1 + \frac{m_\alpha}{m_{\text{daughter}}} \right] \]

which is the desired result.

From example 27.2, \( Q = 4.28 \text{ MeV}, m_\alpha = 4.0015\text{u}, m_{\text{daughter}} = m_{\text{thorium}} = 233.99409\text{u}, \) and we find the velocity of the alpha particle to be \( 1.42 \times 10^7 \text{ m/s} \).

**Tuesday, March 10, 2009**

**Chapter 21**

**Questions**

26.7 \( \alpha \) particles are low energy so they do not penetrate very far into tissue. They are stopped by the skin producing burns to the exposed patch.

\( \beta \) particles are higher energy and will penetrate farther into tissue than \( \alpha \) particles. Beta particles travel about 100 times farther than alpha particles. Whereas alpha particles seldom pass beyond the outer, dead layer of the skin, the free, fast-moving electrons and positrons that constitute beta radiation penetrate for about a quarter of an inch into living matter.

\( \gamma \) rays are very high energy photons and are not stopped by the skin. They pass through almost undisturbed. However they will ionize atoms as they pass by. Gamma rays and X rays will pass readily through a large organism; they reach the innermost recesses of the body and injure highly sensitive tissues, but they produce only about one twentieth of the damage inflicted on cells by alpha particles.

26.8 To determine the unknown product we use both conservation of mass and charge.

a) For the following reaction \( ^{60}_{27}\text{Co} \rightarrow ^{60}_{28}\text{Ni} + ^A_ZX \) conservation of charge gives me \( 27 = 28 + Z' \) which gives \( Z' = -1 \). Conservation of mass gives me \( 60 = 60 + A' \) which gives \( A' = 0 \). Thus the unknown particle is a beta particle, \( ^{-1}_0e = ^{A'}_ZX \).

b) For the following reaction \( ^A_ZX \rightarrow ^{234}_{91}\text{Pa} + ^0_{-1}e \) conservation of charge gives me \( Z' = 91 - 1' \) which gives \( Z' = 90 \). Conservation of mass gives me \( A' = 234 + 0 \) which gives \( A' = 234 \). Thus the unknown particle is a thorium nucleus, \( ^{234}_{90}\text{Th} = ^{A'}_{Z'}X \).

c) For the following reaction \( ^A_ZX \rightarrow ^{234}_{90}\text{Th} + ^4_2\text{He} \) conservation of charge gives me \( Z' = 90 + 2 \) which gives \( Z' = 92 \). Conservation of mass gives me \( A' = 230 + 4 \).
which gives \( A' = 234 \). Thus the unknown particle is a uranium nucleus, \( ^{234}\text{U} \rightarrow A'X \).

26.10 No it does not matter when each experimenter starts their respective experiments. Since the radioactive decay law can, for example, be written in terms of the activity of the radioactive sample, as long as the experimenters know their initial sample activity then the decay constant can be determined. The decay constant will be the same for both experimenters and thus the half-life can be calculated independent of the initial activity the experimenters started with.

26.16 Positrons are given off during the decay of the nuclei of specific radioisotopes. When matter collides with its corresponding antimatter, both are annihilated. When a positron meets an electron, the collision produces two gamma rays having the same energy, but going in opposite directions. The gamma rays leave the patient’s body and are detected by the PET scanner. The information is then fed into a computer to be converted into a complex picture of the patient’s working brain generally. The detection process creates two \( \gamma \)-ray photons with the exact same energy (0.511 MeV) at nearly 180° apart in a line. This coincidence measurement gives us the signature of positron-electron annihilation.

Multiple-Choice
26.9 D
26.10 A
26.11 D

Problems
26.13 The Chernobyl accident
a. The half-life is the time needed for \( \frac{1}{2} \) of the radioactive nuclei to disintegrate and we can relate the half-life to the decay constant. We have
\[
N = \frac{N_0}{2} = N_0 e^{-\frac{\lambda t}{2}} \rightarrow t \approx \frac{\ln 2}{\lambda} \quad \text{and the decay constant is therefore}
\]

b. To calculate the storage time, we use the radioactive decay law with \( N = 0.15N_0 \). Therefore
\[
0.15N_0 = N_0 e^{-\left(0.0866\text{days}^{-1}\right)t} \rightarrow t = \frac{\ln(0.15)}{-0.0866\text{days}^{-1}} = 21.9\text{days}.
\]

26.15 We first determine the decay constant for palladium and we have
\[
\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{17\text{days}} = 0.041\text{days}^{-1}. \quad \text{Next we calculate the number of nuclei that remain after the 30 days from the radioactive decay law and find}
\]
\[
N = N_0 e^{-\lambda t} = N_0 e^{-0.041\text{days}^{-1} \times 30\text{days}} = 0.292N_0 \quad \text{Therefore the number that have decayed is}
\]
\[
N_{\text{decayed}} = 1 - 0.292N_0 = 0.708N_0. \quad \text{Further we know how much energy we need to destroy the tumor and we know the energy of the emitted gamma rays. Thus we can determine how many initial atoms we need. This produces}
\]
2.12J = 0.708N_o \times 21000eV \times \frac{1.6 \times 10^{-19}J}{1eV} \rightarrow N_o = 8.92 \times 10^{14}. \text{ Now that we know the initial number of radioactive atoms we can calculate the initial activity of the palladium and we find } A_o = \lambda N_o = 0.041 \text{days}^{-1} \times 8.92 \times 10^{14} = 3.66 \times 10^{13} \text{day}^{-1}. \text{ Lastly the initial mass of palladium can be calculated from }

m = N_o m_{pd} = 8.92 \times 10^{14} \times 103u \times \frac{1.66 \times 10^{-27} \text{kg}}{1u} = 1.53 \times 10^{-10} \text{kg} = 15.3\text{ng}.

26.16 Uranium Decay
a. \text{ }^{238}_{92}U \rightarrow \frac{3}{2}\text{He} + \text{ }^{234}_{90}Th
b. \text{ Thorium-90}
c. Q = (m_U - m_\alpha - m_{Th})c^2 = (238.00018u - 4.00150u - 233.99409u)c^2 = 0.00459u \times \frac{931.5\text{MeV}}{u^2} \times c^2 = 4.276\text{MeV}
d. KE = (\gamma - 1)m_\alpha c^2 \rightarrow 4.276\text{MeV} = (\gamma - 1) \times \left(4.00150u \times \frac{931.5\text{MeV}}{u^2}\right)\times c^2 = 3727.4\text{MeV} \rightarrow \gamma = 1.00115 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \text{ Solving for } v \text{ we obtain } v = 0.048c.
e. \text{ Since the speed of the alpha particle is less than 0.1c the alpha particle is non-relativistic.}

26.17 Here the initial mass } M_0 = 1 \text{ and the final mass is 0.01% of the initial mass. Thus } M_f = 1 \times 10^{-4} M_0. \text{ Referring to problem #7, the decay constant } \lambda = 0.024 \text{ yr}^{-1}. \text{ From the radioactive decay law: }

M_f = M_0 e^{-\lambda t} \rightarrow 1 \times 10^{-4} M_0 = M_0 e^{-\left(0.024\text{yr}^{-1}\right)t} \rightarrow t = 383.4\text{ yrs}.

26.19 The activity when the bone chip is measured and is 0.5 decay/sec. The initial activity when the animal died needs to be determined. In the bone there is found 5g of carbon. Since 1 mole of carbon contains 6.02x10^{23} atoms and 1 mole of carbon has a mass of 12 g, there are 2.51x10^{23} carbon nuclei. Further the ratio of ^{14}\text{C}/^{12}\text{C} has remained relatively constant and has a value of 1.3x10^{12}. Thus the number of ^{14}\text{C} nuclei is given as (1.3x10^{12})(2.51x10^{23} \text{ nuclei}) = 3.26x10^{11} - ^{14}\text{C} nuclei when the animal died. The initial activity is a product of the decay constant and the number of ^{14}\text{C} nuclei present when the animal died. The decay constant is found from the half-life of carbon (5730yrs).

\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{5730\text{yrs}} \times \frac{1\text{yr}}{3.2 \times 10^{7} \text{sec}} = 3.78 \times 10^{-12} \text{sec}^{-1}. \text{ The initial activity is }

\lambda N = (3.78x10^{-12} \text{ sec}^{-1})(3.26x10^{11} \text{ ^{14}\text{C} nuclei}) = 1.23 \text{ Bq}. \text{ To calculate the age of the bone we use the radioactive decay law }

A = A_0 e^{-\lambda t} \rightarrow 0.5Bq = 1.23Bq e^{-\left(3.78x10^{-12} \text{ sec}^{-1}\right)t} \rightarrow \ln\left(\frac{0.5}{1.23}\right) = -(3.78 \times 10^{-12} \text{ sec}^{-1})t \rightarrow -0.90 = -(3.78 \times 10^{-12} \text{ sec}^{-1})t \rightarrow t = 2.38 \times 10^{11} \text{ sec} = 7438\text{yrs}.\)
Since the bone is only about 7438 years old and knowing that the dinosaurs disappeared over 65 million years ago, it is probably not the bone of a dinosaur.