Physics 111 Homework Solutions Week #9

Wednesday, February 25, 2009
Chapter 22
Questions

22.2  As the spray is adjusted from a jet to a spray the exit aperture is decreasing.

22.6  To show that the two angles in figure 23.10 are equal consider the following diagram.

Since the distance from the screen to the slits (D) is much greater than the separation (d) of the slits. The rays that leave from the two slits are essentially parallel to one another thus making an angel \( \theta \) with the normal to the slit opening. Moving ray 1 down toward ray 2 parallel to itself until it intersects ray 2 we define the difference in path, \( \delta \) between ray 2 and ray 1 as shown. This produces the two right angles shown. Thus \( \alpha + \theta = 90^\circ \) and the angle between the normal to the slits and the slits is also \( 90^\circ \), therefore the angle in question is \( 90^\circ - \alpha = \theta \) as is required. Hence \( \delta = d \sin \theta \).

22.12  With regard to a diffraction grating, \( d \) is the spacing between the grooves on the grating. For a grating with 10,000 grooves/cm, the spacing between the grooves is given as \( (1/10000) \) cm/groove, which gives \( 1 \times 10^{-4} \) cm between each groove.
Multiple-Choice
- None

Problems
22.6 For this Young’s double slit experiment, since $D \gg d$, we can use the small angle approximation for $\sin \theta$. Thus $\sin \theta \approx \tan \theta = \frac{y_m}{D}$ and therefore,

$$d \sin \theta \sim d \tan \theta = d \frac{y_m}{D} = m \lambda$$

and the spacing between adjacent fringes is given by:

$$\frac{\Delta y_m}{\Delta m} = \frac{\lambda D}{d} = \frac{(633 \times 10^{-9} \text{m})(3.5 \text{m})}{0.12 \times 10^{-3} \text{m}} = 0.0185 \text{m} = 18.5 \text{mm}.$$ 

22.10 A double-slit experiment with 550nm light

a. The fringe spacing is given as $\frac{\Delta y_m}{\Delta m}$, where for small angles we have $d \frac{y_m}{D} = m \lambda$. For $\Delta m = 1$, the spacing is

$$\frac{\Delta y_m}{\Delta m} = \frac{\lambda D}{d} = \frac{(550 \times 10^{-9} \text{m})(4 \text{m})}{0.14 \times 10^{-3} \text{m}} = 0.0157 \text{m} = 15.7 \text{mm}$$

b. The width of the diffraction pattern is given by $\sin \theta = \frac{\lambda}{a} = \frac{y_D}{D}$, which is valid for small angles. Thus we have for the $\frac{1}{2}$ of the central width

$$y_D = \frac{\lambda D}{a} = \frac{(550 \times 10^{-9} \text{m})(4 \text{m})}{0.02 \times 10^{-3} \text{m}} = 0.11 \text{m} = 110 \text{mm}$$

and therefore the total width is twice this value, or 220mm.

c. The number of fringes about the central maximum is given by $\frac{220 \text{mm}}{15.7 \text{mm}} = 14$.

Including the central maximum produces 15 total fringes. However, the last two are at the diffraction minimum and are not visible. Thus the number of visible fringes is 13.

d. In water the wavelength of light changes. The wavelength in water is given by

$$\lambda_{\text{water}} = \frac{\lambda_{\text{air}}}{n_{\text{water}}} = \frac{550 \times 10^{-9} \text{m}}{1.33} = 4.13 \times 10^{-7} \text{m} = 413 \text{nm}.$$ Redoing parts a – c gives:

a. The fringe spacing is given as $\frac{\Delta y_m}{\Delta m}$, where for small angles we have

$$d \frac{y_m}{D} = m \lambda.$$ For $\Delta m = 1$, the spacing is

$$\frac{\Delta y_m}{\Delta m} = \frac{\lambda D}{d} = \frac{(413 \times 10^{-9} \text{m})(4 \text{m})}{0.14 \times 10^{-3} \text{m}} = 0.0118 \text{m} = 11.8 \text{mm}$$

b. The width of the diffraction pattern is given by $\sin \theta = \frac{\lambda}{a} = \frac{y_D}{D}$, which is valid for small angles. Thus we have for the $\frac{1}{2}$ of the central width
\[
\gamma_D = \frac{\lambda D}{a} = \frac{(413 \times 10^{-9} \text{ m})(4m)}{0.02 \times 10^{-3} \text{ m}} = 0.0826 \text{ m} = 82.6 \text{ mm}
\]
and therefore the total width is twice this value, or 165.2 mm.

c. The number of fringes about the central maximum is given by
\[
\frac{165.2 \text{ mm}}{11.87 \text{ mm}} = 14.
\]
Including the central maximum produces 15 total fringes. However, the last two are at the diffraction minimum and are not visible. Thus the number of visible fringes is 13.

e. If a polarizer is placed in front of each of the slits with their transmission axes at right angles to each other then supposing that the sources are vertically polarized, one of the light beams will be extinguished while the other is transmitted. You will see no interference pattern, only a single slit diffraction pattern with a bright central spot and ever decreasing intensities as you move away from the central maximum.

Friday, February 27, 2009
Chapter 22
Questions
- None

Multiple-Choice
22.3 A
22.4 C
22.5 B
22.6 B
22.7 B
22.9 D

Problems
22.7 In this double slit experiment we are given a wavelength of light of 488 nm, a slit width for each slit of 0.8 mm and a slit spacing of 0.24 mm. The angular distance from the central maximum to the 1st interference maximum is given by
\[
d \sin \theta = m \lambda, \text{ with } m = 1, \text{ so } \theta = \sin^{-1}\left(\frac{488 \times 10^{-9} \text{ m}}{0.24 \times 10^{-3} \text{ m}}\right) = 0.177^\circ.
\]
The angular distance from the central maximum to the 1st diffraction minimum is given by
\[
\sin \theta = \frac{\lambda}{a} = \frac{488 \times 10^{-9} \text{ m}}{0.08 \times 10^{-3} \text{ m}} = 0.0061 \rightarrow \theta = 0.350^\circ.
\]
Thus in the 1st diffraction minimum there will be \( \frac{0.350}{0.117} = 3 \) interference fringes to the right and to left of the central. Including the central maximum we arrive at a total of 7 fringes. However the last two lie at the diffraction minimum and will not be visible, so we can only see 5 fringes.
22.8 A double-slit experiment with 500nm light
In this double-slit experiment we are given a wavelength of light of 500nm and a slit width for each slit of 0.1mm.

a. The spacing between the slits is given by:
   \[ d = \frac{\Delta m}{\Delta y_m} D \lambda = \left(\frac{1}{5 \times 10^{-3} m}\right) (4m) (500 \times 10^{-9} m) = 4 \times 10^{-4} m = 0.4 mm. \]

b. The angular distance from the central maximum to the 1st diffraction minimum is given by \( \sin \theta = \frac{\lambda}{a} \), and the distance is given as
   \[ y = D \sin \theta = D \frac{\lambda}{a} = (4m) \frac{500 \times 10^{-9} m}{0.1 \times 10^{-3} m} = 0.02 m = 20 mm. \]

c. The distance between bright interference fringes is given as
   \[ d \sin \theta = D \frac{\lambda}{a} = \Delta y_m \Delta m. \]
   and therefore
   \[ \Delta y_m = D \frac{\lambda}{d} = 4m \frac{500 \times 10^{-9} m}{0.4 \times 10^{-3} m} = 0.005 m = 5 mm. \]

In the 1st diffraction minimum there are \( \frac{20 mm}{5mm} = 4 \) interference fringes on either side of the central maximum for a total of 8 fringes. In addition we need to add in the central maximum for a total of 9 fringes. However, the two extreme interference maxima are located at the 1st diffraction minima and are not visible, thus we need to subtract 2. The total number of visible interference fringes is 7.

22.9 Double-slit diffraction with a beam of electrons
In this double-slit experiment we are given a beam of electrons incident on a two slits each of width of 0.1 \( \mu m \) and separated by 4.0 \( \mu m \).

a. In order to calculate the de Broglie wavelength for the electrons we need to calculate their momentum and hence their velocity. Accelerating the electron through a potential difference of 100V does work on the electrons and the electrons acquire a speed given by:
   \[ v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2(1.6 \times 10^{-19} C)(100V)}{9.11 \times 10^{-31} kg}} = 5.93 \times 10^6 \frac{m}{s}. \]
   Their de Broglie wavelength is given as
   \[ \frac{\hbar}{p} = \frac{\hbar}{mv} = \frac{6.63 \times 10^{-34} Js}{(9.11 \times 10^{-31} kg)(5.93 \times 10^6 \frac{m}{s})} = 1.22 \times 10^{-10} m. \]

b. The width of the central diffraction maximum is twice the distance from the central maximum to the 1st diffraction minimum on either side, calling it \( y_1 \). This distance is given as:
   \[ y_1 = D \sin \theta = D \frac{\lambda}{a} = 20m \frac{1.22 \times 10^{-10} m}{1 \times 10^{-6} m} = 2.4 \times 10^{-3} m = 2.4 mm. \]
Therefore the total width is twice this distance or 4.8mm.
c. To calculate the number of fringes, we will use the same logic as in problem 8c. The distance between adjacent bright interference fringes is given as:

$$\frac{\Delta y_m}{\Delta m} = \frac{\lambda D}{d} = \frac{1.22 \times 10^{-10} m \cdot 20m}{4 \times 10^{-6} m} = 6.1 \times 10^{-4} m = 0.61 mm.$$ 

The distance from the central maximum to the 1st diffraction minimum is ½ of the width found in part 9b. Thus the number of fringes around the central maximum is

$$2 \times \frac{2.44 mm}{0.61 mm} = 8 \text{ fringes}$$

plus the central maximum for a total of 9 fringes. The two extreme fringes are at a diffraction minimum and are not visible. Thus there is a total of 7 fringes visible.

22.17 A double slit experiment with protons

a. The wavelength is given by the de Broglie relation. We have

$$\lambda = \frac{h}{p} = \frac{h}{mv},$$

where the velocity is determined from the potential difference that the protons were accelerated though. Accelerating the protons through a potential difference does work on the protons and this work changes their kinetic energy and hence their speed. The speed is given as

$$W = q \Delta V = \Delta KE = \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$$

and therefore the wavelength of the proton is

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34} Js}{1.67 \times 10^{-27} kg \times 6.9 \times 10^5 \frac{m}{s}} = 5.7 \times 10^{-13} m.$$ 

b. The center-to-center spacing of the constructive interference maxima is, assuming that the small angle approximation is valid,

$$d \sin \theta_m \sim d \tan \theta_m = d \frac{y_m}{D} = m \lambda$$

$$\rightarrow \Delta y_m = \frac{\Delta m \lambda D}{d} = \frac{1 \times 5.7 \times 10^{-13} m \times 20m}{4 \times 10^{-6} m} = 2.9 \times 10^{-6} m = 2.8 \mu m$$

c. The width of ½ of the central maximum is given as

$$a \sin \phi_m \sim a \tan \phi_m = a \frac{y_m}{D} = m' \lambda$$

$$\rightarrow \Delta y_m' = \frac{\Delta m' \lambda D}{a} = \frac{1 \times 5.7 \times 10^{-13} m \times 20m}{1 \times 10^{-6} m} = 11.4 \times 10^{-6} m = 11.4 \mu m$$

Therefore the full width is twice this value or 22.8 \mu m.

d. The number of interference fringes is \frac{22.8 \mu m}{2.8 \mu m} = 8. Including the central maximum produces 9 total fringes. However, the last two are at the
diffraction minimum and are not visible. Thus the number of visible fringes is 7.

Monday, March 2, 2009
Chapter 24
Questions
24.1 Based on special relativity we know that as a particle with mass travels near the speed of light its mass increases. In order to accelerate this object from rest to a speed near that of light would require an ever increasing force (one that rapidly becomes larger by a factor of $\gamma$.) There are no known forces that could accelerate a particle with mass to the speed of light in a finite amount of time and with a finite amount of energy. So for objects with no rest mass, as they travel at the speed of light, there mass does not increase with increasing speed and we avoid these problems of accelerating the massless particles.

24.6 The Compton shift in wavelength for the proton and the electron are given by $\Delta \lambda_p = \frac{\hbar}{m_p c} (1 - \cos \phi)$ and $\Delta \lambda_e = \frac{\hbar}{m_e c} (1 - \cos \phi)$ respectively. Evaluating the ratio of the shift in wavelength for the proton to the electron, evaluated at the same detection angle $\phi$, we find $\frac{\Delta \lambda_p}{\Delta \lambda_e} = \frac{m_e}{m_p} = \frac{9.11 \times 10^{-31} \text{kg}}{1.67 \times 10^{-27} \text{kg}} = \frac{1}{1833} = 5 \times 10^{-4}$. Therefore the shift in wavelength for the proton is smaller than the wavelength shift for the electron.

Multiple-Choice
- None

Problems
24.1 Relativistic energy and momentum for an object of mass $m$.
For an object with a $m = 1 \text{kg}$ rest mass, it has a rest energy of $E = mc^2 = 9 \times 10^{16} \text{ J}$.
The Lorentz factor is given by: $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$, with the relativistic momentum $p = \gamma mv$, and the relativistic energy $E^2 = p^2 c^2 + m^2 c^4$.

a. For a velocity of $0.8c$, $\gamma = \frac{1}{\sqrt{1 - \frac{0.8^2}{1}}} = \frac{1}{\sqrt{1 - 0.64}} = 1.67$. The relativistic momentum and energy are therefore,

\[ p = \gamma mv = 1.67 \times 1 \text{kg} \times 0.8 \times 3 \times 10^8 \frac{\text{m}}{\text{s}} = 4.0 \times 10^8 \frac{\text{km}}{\text{s}} \]
and
\[ E = \sqrt{p^2 c^2 + m^2 c^4} = \sqrt{\left(4.0 \times 10^8 \frac{\text{km}}{\text{s}} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 + (9 \times 10^{16} \text{ J})^2} = 1.5 \times 10^{17} \text{ J} \]
b. Following the procedure in part a, for a velocity of 0.9c, 
\[ \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.81}} = 2.29 \]
the relativistic momentum is \(6.19 \times 10^8\) kgm/s, and the relativistic energy is \(2.07 \times 10^{17}\) J.

c. For a velocity of 0.95c, \(\gamma = 3.20\), the relativistic momentum is \(9.13 \times 10^8\) kgm/s, and the relativistic energy is \(2.88 \times 10^{17}\) J.

d. For a velocity of 0.99c, \(\gamma = 7.09\), the relativistic momentum is \(2.11 \times 10^9\) kgm/s, and the relativistic energy is \(6.387 \times 10^{17}\) J.

e. For a velocity of 0.999c, \(\gamma = 22.4\), the relativistic momentum is \(6.70 \times 10^9\) kgm/s, and the relativistic energy is \(2.0 \times 10^{18}\) J.

24.2 For an electron with rest mass \(9.11 \times 10^{-31}\) kg, it has a rest energy of \(E = mc^2 = 8.199 \times 10^{-14}\) J = 0.511 MeV. The Lorentz factor is given by: \(\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}\), and the relativistic momentum and energy are given respectively as \(p = \gamma mv\) and \(E = \sqrt{p^2 + m^2c^4}\).

a. For the electron with a velocity of 0.8c, \(\gamma = 1.67\). The relativistic momentum and energy are therefore, 
\[ p = \gamma mv = 1.67 \times 9.11 \times 10^{-31} \text{ kg} \times 0.8 \times 3 \times 10^8 \text{ m/s} = 3.65 \times 10^{-22} \text{ kgm/s} \quad \text{and} \]
\[ E = \sqrt{p^2 + m^2c^4} = \left(3.65 \times 10^{-22} \frac{\text{kgm}}{\text{s}} \times 3 \times 10^8 \frac{\text{m}}{\text{s}}\right)^2 + \left(8.199 \times 10^{-14} \frac{\text{J}}{\text{s}}\right)^2 = 1.37 \times 10^{-13} \text{ J} \]
Using the fact that \(1.6x10^{19}\) J = 1 eV, the relativistic energy is given as 
\[ E = 1.37 \times 10^{-13} \text{ J} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 0.855 \times 10^6 \text{ eV} = 0.855 \text{ MeV} \]
Further it can be shown that the total relativistic energy is given as \(E = \gamma mc^2 = \gamma E_{\text{rest}}\). Thus the relativistic energy can be computed in a more efficient method. 
\[ E = \gamma E_{\text{rest}} = 1.67 \times 0.511 \text{ MeV} = 0.855 \text{ MeV} \]

b. Following the method outlined in part a, for a velocity of 0.9c, \(\gamma = 2.29\). The relativistic momentum is therefore, 
\[ p = \gamma mv = 2.29 \times 9.11 \times 10^{-31} \text{ kg} \times 0.9 \times 3 \times 10^8 \text{ m/s} = 5.63 \times 10^{-22} \text{ kgm/s} \quad \text{and} \]
\[ E = \gamma E_{\text{rest}} = 2.29 \times 0.511 \text{ MeV} = 1.17 \text{ MeV} \]

c. For a velocity of 0.95c, \(\gamma = 3.2\). The relativistic momentum is therefore, 
\[ p = \gamma mv = 3.2 \times 9.11 \times 10^{-31} \text{ kg} \times 0.95 \times 3 \times 10^8 \text{ m/s} = 8.31 \times 10^{-22} \text{ kgm/s} \quad \text{and} \]
\[ E = \gamma E_{\text{rest}} = 3.2 \times 0.511 \text{ MeV} = 1.63 \text{ MeV} \]

d. For a velocity of 0.99c, \(\gamma = 7.09\). The relativistic momentum is therefore, 
\[ p = \gamma mv = 7.09 \times 9.11 \times 10^{-31} \text{ kg} \times 0.99 \times 3 \times 10^8 \text{ m/s} = 1.92 \times 10^{-21} \text{ kgm/s} \quad \text{and} \]
\[ E = \gamma E_{\text{rest}} = 7.09 \times 0.511 \text{ MeV} = 3.62 \text{ MeV} \]

e. For a velocity of 0.999c, \(\gamma = 22.4\). The relativistic momentum is therefore,
\[
p = \gamma m v = 22.4 \times 9.11 \times 10^{-31} \text{kg} \times 0.999 \times 3 \times 10^8 \text{ m/s} = 6.12 \times 10^{-21} \text{ kg m/s} \quad \text{and}
\]
\[
E = \gamma E_{\text{rest}} = 22.4 \times 0.511 \text{MeV} = 11.45 \text{MeV}.
\]

24.4 The relativistic momentum and relativistic energy are given as \( p = \gamma m v \) and \( E = \gamma m c^2 \) respectively. To show the relation between energy and momentum, equation (24.8), we start by squaring the relativistic energy. This gives us \( E^2 = \gamma^2 m^2 c^4 \). Next, we use a mathematical “trick.” We add and subtract the same quantity from the right hand side of the equation we just developed. The quantity we want to add and subtract is \( v^2 \). This produces factoring out a factor of \( c^2 \), \( E^2 = \gamma^2 m^2 c^2 \left(c^2 + v^2 - v^2 \right) \). Expanding this result we get \( E^2 = \gamma^2 m^2 c^2 v^2 + \gamma^2 m^2 c^2 \left(c^2 - v^2 \right) \). Recognizing that the first term is nothing more than \( p^2 c^2 \) allows us to write \( E^2 = p^2 c^2 + \gamma^2 m^2 c^2 \left(c^2 - v^2 \right) \). Factoring out a \( c^2 \) from the 2\textsuperscript{nd} term on the right hand side give us \( E^2 = p^2 c^2 + \gamma^2 m^2 c^4 \left(1 - \frac{v^2}{c^2} \right) \).

The quantity \( \left(1 - \frac{v^2}{c^2} \right) \) is simply \( \frac{1}{\gamma^2} \). Therefore we arrive at the desired result, \( E^2 = p^2 c^2 + m^2 c^4 \).

24.7 For a \( 1.2 \times 10^6 \text{eV} \times \frac{1.6 \times 10^{-19} \text{J}}{1 \text{eV}} = 1.92 \times 10^{-13} \text{ J} \) photon,

a. its momentum is given by \( p = \frac{E}{c} \frac{1.92 \times 10^{-13} \text{J}}{3 \times 10^8 \text{ m/s}} = 6.4 \times 10^{-22} \text{ kg m/s} \).

b. its wavelength is given by the de Broglie relation \( \lambda = \frac{\hbar}{p} = \frac{6.63 \times 10^{-34} \text{ Js}}{6.4 \times 10^{-22} \text{ kg m/s}} = 1.04 \times 10^{-12} \text{ m} \).

c. its frequency is given by \( f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{1.04 \times 10^{-12} \text{ m}} = 2.9 \times 10^{20} \text{ s}^{-1} \).

24.13 The relativistic energy is given as \( E_{\text{rel}} = \sqrt{p^2 c^2 + m^2 c^4} \). The rest energy, \( m c^2 \) is \( 8.199 \times 10^{-19} \text{ J} \) \( = 0.511 \text{ MeV} \). From the de Broglie wavelength we can calculate the momentum of the electron, \( p = \frac{\hbar}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{0.0012 \times 10^{-9} \text{ m}} = 5.525 \times 10^{-22} \text{ kg m/s} \). Thus the relativistic energy is

\[
E_{\text{rel}} = \sqrt{(5.525 \times 10^{-22} \text{ kg m/s})^2 \left(3 \times 10^8 \text{ m/s} \right)^2 + (8.199 \times 10^{-19} \text{ J})^2} = 1.657 \times 10^{-13} \text{ J} = 1.04 \text{ MeV}.
\]

Tuesday, March 3, 2009
Chapter 21
Questions
- None

Multiple-Choice
- None

Problems
24.11  A Compton Effect experiment

a. The wavelength and momentum of the incident gamma ray are given as
\[ E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{(6.63 \times 10^{-34} \text{ Js})(3 \times 10^8 \text{ m/s})}{1.6 \times 10^6 \text{ eV} \times \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}}} = 7.77 \times 10^{-13} \text{ m} \text{ and} \]
\[ p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} \text{ Js}}{7.77 \times 10^{-13} \text{ m}} = 8.53 \times 10^{-22} \frac{\text{kg m}}{\text{s}} \text{ respectively.} \]

b. Using the Compton formula for the wavelength of the scattered photon and the fact that the energy is inversely proportional to the wavelength we can write
\[ \lambda' = \lambda + \frac{h}{m_e c} (1 - \cos \phi) \rightarrow \frac{\lambda'}{hc} = \frac{\lambda}{hc} + \frac{1}{m_e c^2} (1 - \cos \phi) \rightarrow \frac{1}{E'} = \frac{1}{E} + \frac{(1 - \cos \phi)}{m_e c^2} \]

. 

c. The energy of the scattered gamma ray photon is
\[ \frac{1}{E'} = \frac{1}{1.6 \text{ MeV}} + \frac{(1 - \cos 50)}{0.511 \text{ MeV}} = 1.324 \text{ MeV}^{-1} \rightarrow E' = 0.755 \text{ MeV} \text{ where the rest mass of the electron is given as} \]
\[ m_e c^2 = \left(9.11 \times 10^{-31} \text{ kg} \right) \left(3 \times 10^8 \text{ m/s} \right)^2 \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 0.511 \text{ MeV} . \]

d. The kinetic energy of the recoiling electron is found using conservation of energy where when the incident gamma ray photon interacts with electrons in the sample, the gamma ray photon loses some energy to the electron as it scatters. Thus the kinetic energy of the recoiling electron is
\[ E_{\text{incident}} = E_{\text{scattered}} + KE_e \]
\[ \therefore KE_e = E_{\text{incident}} - E_{\text{scattered}} = 1.6 \text{ MeV} - 0.755 \text{ MeV} = 0.845 \text{ MeV} \]

e. The speed of the recoiling electron is given by using the expression for the relativistic kinetic energy. We have
\[ KE = 0.845 \text{ MeV} = (\gamma - 1)m_e c^2 \rightarrow (\gamma - 1) 0.511 \text{ MeV} \]
\[ \gamma = 1 + \frac{0.845 \text{ MeV}}{0.511 \text{ MeV}} = 1 + 1.654 = 2.654 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \rightarrow v^2 = 0.858 c^2 \rightarrow v = 0.926 c \]

24.12  X-rays on a foil target
a. The energy is given by

\[
E = \frac{hc}{\lambda} = \frac{\left(6.63 \times 10^{-34} \text{ Js}\right) \left(3 \times 10^8 \text{ m/s}\right)}{0.012 \times 10^{-9} \text{ m}} = 1.66 \times 10^{-14} \text{ J} = 0.104 \text{ MeV}.
\]

b. The scattered wavelength is given by

\[
\lambda_f - \lambda_i = \frac{h}{mc} (1 - \cos \phi) = \frac{2h}{mc} = \frac{2\left(6.63 \times 10^{-34} \text{ Js}\right)}{\left(9.11 \times 10^{-31} \text{ kg}\right) \left(3 \times 10^8 \text{ m/s}\right)} = 4.86 \times 10^{-12} \text{ m}.
\]

Thus, \( \lambda_f = 4.86 \times 10^{-12} \text{ m} + 1.2 \times 10^{-11} \text{ m} = 1.686 \times 10^{-11} \text{ m} \). The energy is given by the formula in part a, and is \( 1.18 \times 10^{-14} \text{ J} = 0.0741 \text{ MeV} \).

c. The energy given to the foil is

\[
\Delta E_{\text{foil}} = E_{\text{incident}} = E_{\text{backscattered}} = 0.104 \text{ MeV} - 0.0741 \text{ MeV} = 0.03 \text{ MeV} = 30 \text{ keV}.
\]