1. A cylinder of moment of inertia $I_1$ rotates about a vertical frictionless axle with angular velocity $\omega_i$. A second cylinder that has moment of inertia $I_2$ and initially not rotating is dropped onto the 1st cylinder. Since friction exists between the two surfaces of the cylinders, they eventually reach the same final angular speed, $\omega_f$.
   a. What is the expression for the magnitude of $\omega_f$?

   $$I_i = I_f \rightarrow I_i \omega_i = (I_1 + I_2)\omega_f \rightarrow \omega_f = \frac{I_i \omega_i}{I_1 + I_2}$$

   b. Show that the kinetic energy of the system decreases in this interaction and calculate the ratio of the final rotational energy to the initial rotational energy.

   $$KE_i = \frac{1}{2} I_1 \omega_i^2$$

   $$KE_f = \frac{1}{2} (I_1 + I_2)\omega_f^2 = \frac{1}{2} \left( \frac{I_1}{I_1 + I_2} \right) \omega_i^2 = \frac{I_i^2}{2 (I_1 + I_2)} \omega_i^2 = \frac{I_i \times KE_i}{I_1 + I_2}$$

   Thus $KE_f < KE_i$ and $\text{ratio} = \frac{KE_f}{KE_i} = \frac{I_1}{I_1 + I_2}$.

   c. Why does the kinetic energy of the system decrease?

   Since KE is not conserved due to friction bringing the two masses to the same final rotational speed, the final KE is less than the initial KE.

2. Alex and Sam are riding on a carousel. Alex rides on a horse at the outer rim of the circular platform, twice as far from the center of the circular platform as Sam, who rides on an inner horse. When the carousel is rotating at constant angular velocity, what is Alex’s angular speed?

   a. Alex’s angular speed is twice Sam’s.
   b. Alex’s angular speed is the same as Sam’s.
   c. Alex’s angular speed is half of Sam’s.
   d. Alex’s angular speed is unable to be determined.