Physics 123 Homework Solutions

Week #4 Unit T Thermodynamics

T3B.2
If the volume change is sufficiently small then the gas pressure change will be constant and is given as \( P = 95\text{kPa} \). A volume change of 1% corresponds to 3\( \text{cm}^3 \) and \( \Delta V = V_f - V_i = 297\text{cm}^3 - 300\text{cm}^3 = -3\text{cm}^3 \). Thus the work done in an isobaric compression is given by \( W = -P\Delta V = (-95\text{kPa})*(-3\text{cm}^3) = 0.29\text{J} \). This result is consistent with the idea that a positive amount of work done means energy flowed into the system.

T3B.6
If the volume of the cylinder is 600\( \text{cm}^3 \) initially and 450\( \text{cm}^3 \) finally, while the initial pressure \( P_i = 80\text{kPa} \), we have for an isothermal process \( P_fV_f = Nk_bT = PV_i \rightarrow P_f = \frac{PV_i}{V_f} = 80\text{kPa} \left( \frac{600\text{cm}^3}{450\text{cm}^3} \right) = 107\text{kPa} \).

T3B.7
Let the initial volume and pressure of the cylinder be 600\( \text{cm}^3 \) and 60\( \text{kPa} \) respectively, while the final volume is 450\( \text{cm}^3 \). For monatomic gas, the adiabatic index \( \gamma = 1.67 \), so the final pressure is given as \( P_fV_f = P_iV_i \rightarrow P_f = P_i \left( \frac{V_i}{V_f} \right)^\gamma = 60\text{kPa} \left( \frac{600\text{cm}^3}{450\text{cm}^3} \right)^{1.67} = 97\text{kPa} \). The pressure has increased as we would have expected in a compression.

T3S.8
Let the initial volume be 800\( \text{m}^3 \) and the initial pressure is that of the atmosphere \( P_i = 1\text{atm} \). The final pressure is given as \( P_f = 0.45P_i \). If the monatomic helium expands adiabatically the final volume is given by \( P_fV_f = P_iV_i \rightarrow V_f = V_i \left( \frac{P_i}{P_f} \right)^\gamma = 800\text{m}^3 \left( \frac{1\text{atm}}{0.45\text{atm}} \right)^{1.67} = 5100\text{m}^3 \), and the final temperature is given as (starting at say 295K) \( T_fV_f^{-1} = T_iV_i^{-1} \rightarrow T_f = T_i \left( \frac{P_i}{P_f} \right)^{\frac{1}{\gamma-1}} = 295K \left( \frac{0.45P_i}{P_i} \right)^{\frac{0.67}{1.67}} = 82K \). This seems pretty cold (Nitrogen liquefies at 77K!) The problem is that the expansion is probably not adiabatic over the entire trip the balloon takes to its final altitude. The balloon probably does heat up some.
T3S.10

a. If the pressure and volume are functions of temperature, then applying the product rule to the ideal gas law we find

\[ \frac{dP}{dT} V + P \frac{dV}{dT} = \frac{d}{dT} (Nk_B T) = Nk_B \rightarrow VdP + PdV = Nk_B dT. \]

b. For any infinitesimal process, the first law of thermodynamics implies that \( dE_{\text{thermal}} = dQ + dW \). For an adiabatic process, \( dQ = 0 \), so \( dE_{\text{thermal}} = dW \). For an ideal gas, \( \frac{dW}{dV} = \frac{f}{2} Nk_B T \rightarrow dE_{\text{thermal}} = \frac{f}{2} Nk_B dT = dW \). For an infinitesimal quasistatic expansion or compression \( dW = -PdV \). Substituting this into the above gives

\[ dW = -PdV = -\frac{f}{2} Nk_B dT. \]

c. So, using the result in part b, we have that \(-\frac{2}{f} PdV = Nk_B dT \), which when plugged into part a becomes

\[ VdP + PdV = Nk_B dT = -\frac{2}{f} PdV \rightarrow 0 = VdP + \gamma PdV. \]

Dividing by \( VdV \) gives

\[ \gamma = \frac{dP}{dV} + \frac{P}{V}. \]

d. Now, multiply by \( V^\gamma \) we get

\[ 0 = \frac{dP}{dV} V^\gamma + \frac{P}{V} \frac{dV}{dV} V^\gamma + \gamma V^{\gamma-1} P = \frac{dP}{dV} V^\gamma + P \frac{d}{dV} \left( V^\gamma \right) = \frac{d}{dV} \left( PV^\gamma \right). \]

or

\[ \frac{d}{dV} \left( PV^\gamma \right) = 0 \rightarrow PV^\gamma = \text{constant}. \]

e. Using the ideal gas law we get

\[ 0 = \frac{d}{dV} \left( PV^{\gamma-1} \right) = \frac{d}{dV} \left( Nk_B TV^{\gamma-1} \right) = Nk_B \frac{d}{dV} \left( TV^{\gamma-1} \right) \] which will be true only if \( TV^{\gamma-1} = \text{constant.} \)