Part I: Multiple-Choice

Circle your answer to each question. Any other marks will not be given credit. Each multiple-choice question is worth 2 points for a total of 20 points.

1. A pacemaker is a simple RC device that is used to deliver precisely timed pulses directly to the heart, and suppose that one sends a pulse to a patient’s heart every time the capacitor charges to 0.25 V. It is desired that the patient receive 75 pulses per minute. If the battery in the pacemaker is rated at 9.0 V, and the capacitor is 110 \( \mu \)F, what should the resistance of the pacemaker be?

   a. 26 k\( \Omega \)  
   b. 121 k\( \Omega \)  
   c. 262 k\( \Omega \)  
   d. 4364 k\( \Omega \)

2. Pacemakers, like the one used in question #1, are designed for long-term use and usually use a lithium-iodine battery. What is the expected lifetime of such a battery if a typical current of 5.6\( \mu \)A is produced and the battery supplies a charge of 0.42 Ampere-hours?

   a. 1.5 years  
   b. 8.7 years  
   c. 22.3 years  
   d. 42.4 years

3. A parallel plate, air filled capacitor is connected to a spring having a force constant \( k \), and plate \( b \) is fixed. They rest on a table top as shown below in the top-down view. If charge \( Q \) is placed on plate \( a \), and charge \( -Q \) is placed on plate \( b \), so that the electric field between the plates is given by, \( E = \frac{Q}{\varepsilon_0 A} \) by how much does the spring expand?

   a. \( x = \frac{Q^2}{\varepsilon_0 k} \)  
   b. \( x = \frac{AQ^2}{k\varepsilon_0} \)  
   c. \( x = \frac{Q}{\varepsilon_0 k} \)  
   d. \( x = \frac{Q^2}{A} \)

4. Two charged particles are a distance \( r \) apart. If the charges on the two particles and the distance between the particles are halved, the force between the particles

   a. decreases by a factor of 2.  
   b. remains the same.  
   c. increases by a factor of 2.  
   d. increases by a factor of 8.
5. A 10 µF capacitor is used to defibrillate the heart. It is charged to 100 V. What power is dissipated through the chest if the energy stored in the defibrillator is released in 10 ms?
   a. 0.05 W   b. 0.5 W   c. 5 W   d. 50 W   e. 500 W

6. Which of the following quantities has units of force? The symbols below have the following meanings: \( A = \text{Ampere}, \ m = \text{meter}, \ C = \text{Coulomb}, \ V = \text{Volt}, \) and \( T = \text{Tesla}. \)
   a. AmT   b. Cm/V   c. CV   d. all of the above   e. none of the above.

7. An electron is accelerated through a region of space where a potential difference of 100 kV exists. How fast is the electron traveling at the end of the accelerating region if it started from rest? \( (M_e = 9.11 \times 10^{-31} \text{ kg}) \)
   a. 1.2 \times 10^{17} \text{ m/s}   b. 1.9 \times 10^8 \text{ m/s}   c. 2.3 \times 10^{15} \text{ m/s}   d. 3.0 \times 10^8 \text{ m/s}   e. 5.9 \times 10^6 \text{ m/s}

8. A circuit has a single battery (with constant potential) and some resistors in it. If the equivalent resistance of the circuit is decreased by one third, what happens to the total current and power dissipated by the entire circuit?
   a. both decrease by a factor of 3   b. both increase by a factor of 3   c. both remain constant   d. both decrease by a factor of 9   e. both increase by a factor of 9

9. Suppose that a charge \(-Q\) is fixed at a given location and that another charge \(q_o\) is able to move. The total energy of \(q_o\) due to \(-Q\) is
   a. constant.   b. increasing with decreasing distance between \(-Q\) and \(q_o\).   c. increasing with increasing distance between \(-Q\) and \(q_o\).   d. depends on the acceleration due to gravity.   e. unable to be determined from the information given.

10. Imagine that you are sitting in a room with your back to the back wall and that a beam of electrons, traveling horizontally from the back wall toward the front wall, is deflected towards your right. The uniform magnetic field that exists in the room is directed towards
   a. the ceiling.   b. the floor.   c. the wall in front of you.   d. the left wall.
Part II: Free Response Problems

Please show all work in order to receive partial credit. If your solutions are illegible no credit will be given. Please use the back of the page if necessary, but number the problem you are working on.

1. Given the circuit shown below, answer the following questions.

![Circuit Diagram]

a. What is the equivalent resistance of the circuit?
   \[ R_{eq} = 4\Omega \]

b. What is the total current supplied by the battery?
   \[ I = 3A \]

c. What is the potential drop across the 4 \( \Omega \) resistor?
   \[ V = 6V \]

d. What is the current in the 12 \( \Omega \) resistor?
   \[ I = 0.5A \]

e. What is the power dissipated across the 3 \( \Omega \) resistor?
   \[ P = 6.75W \]
2. *The Chocolate Crumb Mystery:* Everyone has experienced a shock from reaching out to touch a doorknob. This is an example of an electrostatic discharge or sparking. Explosions ignited by electrostatic discharges are a danger in facilities handling powder or grains. Suppose that an explosion occurred in a chocolate cookie factory when chocolate powder was being blown from sacks through a grounded plastic (PVC) pipe into a silo for storage. Somewhere along this route, two conditions for explosion were met: (1) the magnitude of the electric field became $3.0 \times 10^6 \text{ N/C}$ or greater so that sparking could occur and (2) the energy of a spark was $150 \text{ mJ}$ or greater so that it could ignite the chocolate powder explosively. Suppose that a steam of *negatively* charged chocolate crumbs is blown through a circular pipe with radius $R = 5.0 \text{ cm}$. Assume that the charge is spread throughout the volume of dust in the pipe so that the charge density $\rho$ (charge / unit volume) is constant.

a. If the electric field is given by $E = \frac{\rho r}{2\epsilon_0}$, does the electric field increase or decrease with increasing distance from the center of the pipe, $r$? Does the electric field point radially outward from the center or inward toward the center?

Since $E \propto r$, as $r$ increases so too does $E$. The charge (per volume) inside of the pipe is negative, so the field points radially in towards the center of the pipe.

b. If $\rho = 1.1 \times 10^{-3} \text{ C/m}^3$, what is the maximum magnitude of the electric field and where does it occur? Based on this could sparking occur?

$$E = 3.1 \times 10^6 \frac{\text{N}}{\text{C}}$$

Comparing this to $3.0 \times 10^6 \text{ N/C}$, sparking could likely occur!

c. If the electric potential is given by $V = \frac{\rho}{4\epsilon_0} \left( R^2 - r^2 \right)$, what is the potential difference between the pipe’s outside wall and the center?

At the pipe’s center, $r = 0$ and $V = 77.6 \text{kV}$

At the pipe’s edge, $r = R$ and $V = 0$
d. To investigate where the explosion occurred (either at the site where the workers are loading the chocolate powder or at the silo end) the potential difference of the workers is first measured and found to be 7.0 kV (relative to the ground at 0V.) If each worker is modeled as a capacitor with capacitance $200 \times 10^{-12} \text{ F}$, how much energy is stored in the worker-capacitor? If this worker were to discharge this stored energy by touching a conductor, is this energy enough to cause a spark?

$$E = 4.9 \times 10^{-3} \text{ J} = 4.9 \text{ mJ}$$
No spark probably occurred here since $4.9 \text{ mJ} < 150 \text{ mJ}$.

e. If the dust moves down the pipe at a speed of $v = 2.0 \text{ m/s}$, what is the electric current flowing in the pipe, in terms of $R$, and $v$? Evaluate your expression.

$$I = \rho Av = 1.73 \times 10^{-5} \text{ A}$$

f. Compute the electric power and from this the energy in the form of a spark, if the spark lasted for 0.2 s at the pipes end? Is this where the spark occurred?

$$P = 1.34 W$$
$$\Delta E = 0.268 J = 268 \text{ mJ}$$
Since the energy (268 mJ) is above that needed for a spark, 150 mJ, it is sure likely that a spark might have occurred at the end of the pipe as chocolate powder entered the silo and this caused the explosion.
3. Suppose that a capacitor rated at 10,000 \( \mu \text{F} \) and a resistor rated at 1000 \( \Omega \) are connected together to form a RC circuit and that the capacitor is charged by a battery with potential difference \( V \), such that 100\( \mu \text{C} \) of charge is placed on the plates.

a. What is the time constant for this circuit?

\[ \tau = 10 \text{s} \]

b. How long will it take for the capacitor to lose \( 1/2 \) of its initial charge?

\[ t = 6.93 \text{s} \]

c. What is the time-dependant rate of thermal energy (power) produced in the resistor? (Hint: The resistor will heat up as current is passed through it.)

\[ P = -\frac{V_0^2}{R} e^{-\frac{2t}{RC}} \]

d. At what time dependant rate is stored energy lost by the capacitor during the discharging process? (Hint: NO calculation is needed!) What can you conclude about the energy lost by the capacitor during the discharge and the energy transferred to the resistor as heat?

The energy dissipated by the resistor came from that stored in the capacitor. So the energy (per second) lost by the capacitor was transferred to the resistor in the form of heat. Thus, energy is conserved!

e. What was the potential of the battery used to charge the capacitor?

\[ V_0 = 10mV \]

Multiple-Choice Answers
1. c  5. c  9. a  
2. b  6. a  10. b  
3. a  7. b  
4. b  8. b