BME/ECE 386 EXAM 1 (W19) **NAME:** Solutions

4 problems for 100 pts

Problem #1: Short Answers (25 pts)

(a) Strain Gauge (4 pts): A 350 ohm constantan strain gauge (G = 2.1) experiences a strain ε = -150 μstrain. Does the gauge resistance increase or decrease, and by how much? Express your answer in ohms.

$\frac{\Delta R}{R} = G\varepsilon \rightarrow \Delta R = 2.1 \times (-150 \times 10^{-6}) \times 350\,\Omega$

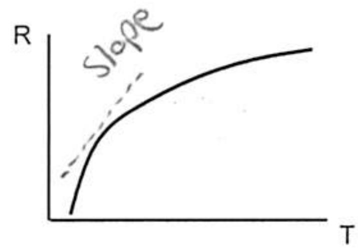
$= \boxed{-0.11\,\Omega}$ Decreases

(b) True or False (2 pts): An instrumentation amplifier should have high input impedance and small common-mode rejection ratio. **If you choose false, explain why.**

False — Large CMRR!

↑ to minimize common mode error

(c) Thermistor (2 pts): Shown on the right is a plot of resistance vs temperature for a thermistor. What is the sign of the tempco?



Slope is positive

→ Positive tempco

(d) Thermocouple (3 pts): A type K thermocouple is made from the junction of alumel and chromel wires. Alumel is an alloy of which two metallic elements? What about chromel?

Alumel = Aluminum + Nickel

Chromel = Chromium + Nickel

(e) True or False (2 pts): An infrared thermometer uses a thermopile detector, which is basically a miniature array of thermistors. **If you choose false, explain why.**

False — Array of thermocouples

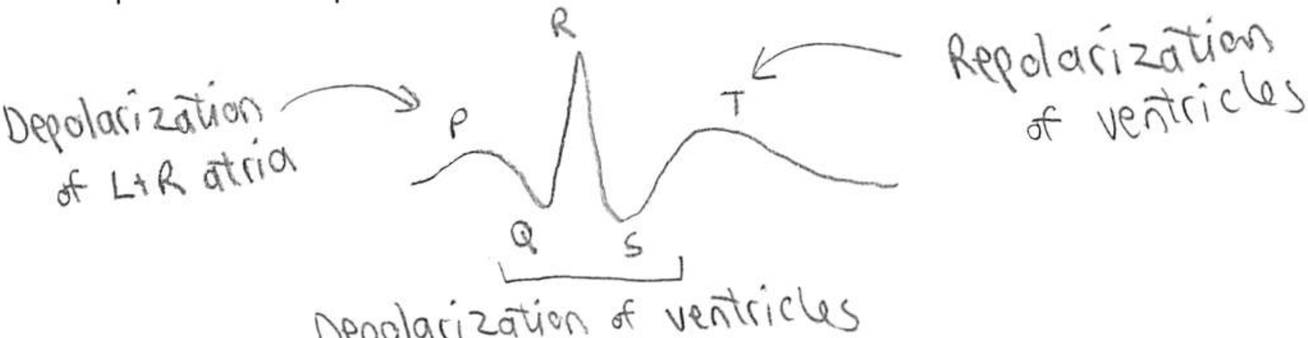
(f) Action Potential (3 pts): The cardiac action potential involves the inward rush of silver ions (depolarization) and phosphorus ions (muscle contraction), followed by the outward rush of chlorine ions (repolarization). **Make corrections, wherever necessary, to the previous statement about the action potential in a cardiac muscle cell.**

Depolarization = inward rush of sodium ions

Muscle contraction = inward rush of calcium ions

Repolarization = outward rush of potassium ions

(g) ECG (3 pts): Sketch a typical ECG waveform and label which portions are due to repolarization or depolarization of a specific section of the heart.



(h) ECG (3 pts): The electrical network of the heart consists of the Sino American node, internasal tracts, Arterial Vehicular node, Bundle of Hers, and Punjabi fibers. **Make corrections, wherever necessary, to the previous statement.**

Sino atrial (SA) node

Internodal Tracts

Atrio ventricular (AV) node

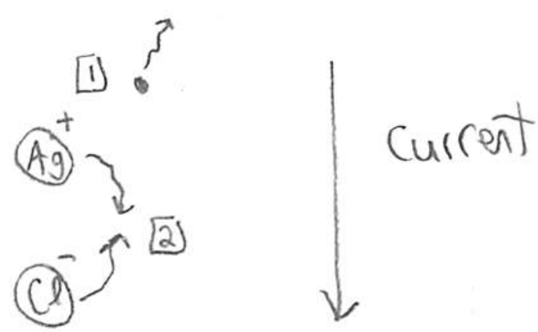
Bundle of His

Purkinje fibers

(i) ECG electrodes (3 pts): In order for current to flow into the body, which chemical reactions occur to silver and silver chloride?

[1] Oxidation of silver

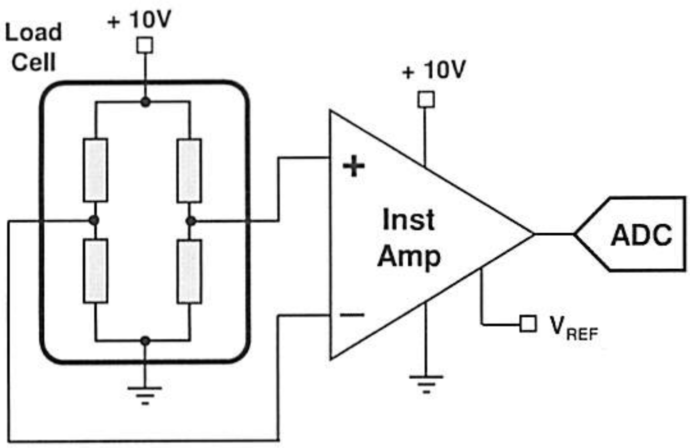
[2] Association of silver chloride

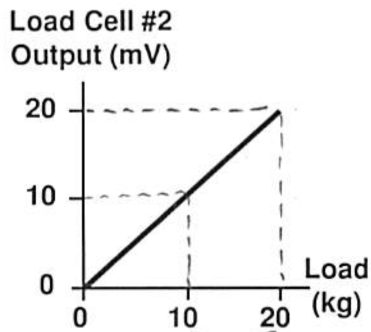


Problem #2: Load Cell (25 pts)

You are asked to design a digital weighing scale. The customer wants to measure a maximum load of 3 kg with a sensitivity of 10 g. The supply voltage is $V_S = +10V$. The instrumentation amplifier has a differential gain Ad = 500, output noise voltage $V_N = 2\ mV_{RMS}$, and $V_{REF} = 2V$. It is powered by +10V and GND, so you can assume the amplifier output is limited to 1V (min) and 9V (max). The ADC has 14 bits (0 to 10V).

You have two load cells available. Load Cell #1 has RO = 1.5 mV/V @ 5 kg. Unfortunately, the RO and L_{RATED} for Load Cell #2 are not known. However, Load Cell #2 has the calibration curve shown in the right figure.





Doesn't really matter which one you choose for L_{rated}

a) Which load cell (there may be more than one) satisfies the design specs? **For each load cell, you must clearly explain why it works or does not work.** Show all work!

b) Suppose your customer wants to replace the original ADC with an Arduino Uno. The Arduino has an ADC with 10 bits (0 to 5V). Can the Arduino be used with one of the available load cells to satisfy both design specs? Show all work! NOTE: You can re-use any relevant results from part (a) (i.e. you do not need to re-do all calculations).

(a) +16

$V_{meas} = A_d \Delta V + V_{ref}$ (← 2V)

$\Delta V = V_s\, RO \frac{L}{L_{rated}}$

Method #1: $V_{meas} < 9V$ @ $L = 3kg$?

LC#1: $V_{meas} = 500 \times 10V \times \frac{.0015V}{V} \times \frac{3kg}{5kg} + 2 = \underline{6.5V}$ ✓

LC#2: $V_{meas} = 500 \times 10V \times \frac{.002V}{V} \times \frac{3kg}{20kg} + 2 = \underline{3.5V}$ ✓

LC#2: $\Delta V = 20mV = 10V \times RO \times \frac{20kg}{20kg}$

$\rightarrow RO = 2\frac{mV}{V}$ @ 20kg

OR $\text{Slope} = \frac{20mV}{20kg} = 1\frac{mV}{kg}$

$\text{Slope} = \frac{\Delta V_{max}}{L_{rated}} = \frac{V_s \times RO}{L_{rated}}$

$\rightarrow RO = 1\frac{mV}{kg} \times \frac{20kg}{10V} = 2\frac{mV}{V}$ @ 20kg

Method #2 $L_{max} > 3kg$ @ $V_{meas} = 9V$?

(extra sheet for work)

LC#1: $L_{max} = \frac{9 - 2V}{500 \times 10V \times \frac{.0015V}{V} \frac{1}{5kg}} = 4.67kg$ ✓

LC#2: $L_{max} = \frac{9 - 2V}{500 \times 10V \times \frac{.002V}{V} \frac{1}{20kg}} = 14.0 kg$ ✓

☆ Must also check sensitivity! $\Delta V_{ADC} = \frac{10 - 0V}{2^{14} - 1} = .61mV$

$V_N = 2mV \leftarrow \Delta V_{min}$

LC#1: $\Delta L_{min} = \frac{2mV}{500 \times 10V \times \frac{1.5mV}{V} \times \frac{1}{5000g}} = 1.33g < 10g$ ✓

LC#2: $\Delta L_{min} = \frac{2mV}{500 \times 10V \times \frac{2mV}{V} \times \frac{1}{20000g}} = 4.0g < 10g$ ✓

☆ Both load cells OK!

(b) Arduino Uno: Max $V_{meas} = 5V$

+9

$\Delta V_{ADC} = \frac{5 - 0}{2^{10} - 1} = 4.9mV$

LC#1: $V_{meas} = 6.5V$ @ 3kg (TOO BIG) X

OR $L_{max} = \frac{5 - 2V}{500 \times 10V \times \frac{.0015V}{V} \times \frac{1}{5kg}} = 2kg$ X (TOO small)

Rule out LC#1

LC#2: $V_{meas} = 3.5V$ @ 3kg ✓

OR $L_{max} = \frac{5 - 2}{500 \times 10 \times .002 \times \frac{1}{20}} = 6kg$ ✓

LC#2 works!

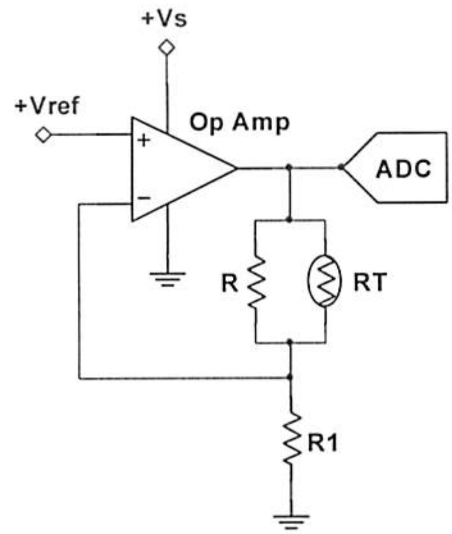
Also, $\Delta L_{min} = \frac{4.9mV}{500 \times 10V \times 2\frac{mV}{V} \times \frac{1}{20000g}} = 9.78g < 10g$ ✓

Problem #3: Temperature (25 pts)

You are asked to design an incubator temperature measurement system that operates from T = 10°C up to T = 50 °C. The magnitude of the sensitivity $|\Delta T_{MIN}|$ must be less than 0.05°C at T = 50 °C.

Consider the thermistor circuit shown to the right. It uses an op amp, a resistor R1, and a thermistor R_T in parallel with a resistor R. The thermistor has the properties shown below:

- **T = 50 °C: R_T = 5.76 kohm α = -3.58 %/°C**
- **T = 10 °C: R_T = 28.51 kohm α = -4.47 %/°C**



The op amp is powered by Vs = +5V and GND. Assume the op amp output is limited to within 1V of each power supply. The output noise voltage is measured to be V_N = 0.5 mV_{RMS}. The ADC has 12 bits and operates from +5V to GND.

a) Use the Golden Rules to show that: $V_{MEAS} = V_{REF} + \frac{V_{REF}}{R1}\left(\frac{R \cdot R_T}{R + R_T}\right)$. Show all work!

b) Assume that V_{REF} = 1V and R1 = 2.7 kohm. The available resistors are R = 10 kohm and 16 kohm. Which resistor (one, both, or none) satisfies all the design requirements? You must explain why a resistor is OK or not OK. Show all work!

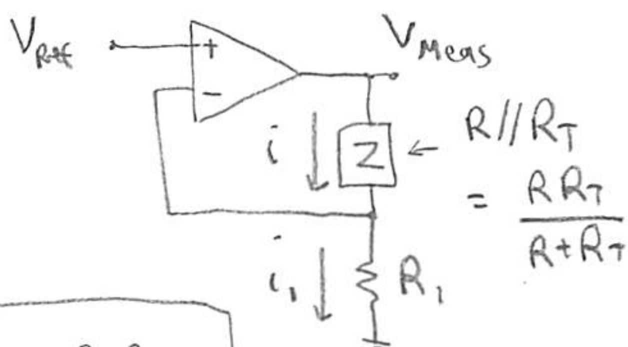
(a) $V_+ = V_- = V_{ref}$

$i = i_1$

$\frac{V_{meas} - V_{ref}}{Z} = \frac{V_{ref} - 0}{R_1}$

$V_{meas} = V_{ref}\frac{Z}{R_1} + V_{ref} = \boxed{V_{ref} + \frac{V_{ref}}{R_1}\left(\frac{R R_T}{R + R_T}\right)}$

(+8)



(b) $1V < V_{meas} < 4V$ $\frac{V_{ref}}{R_1} = \frac{1V}{2.7K} = 0.37 mA$

$T = 50°C$: $R = 10K \rightarrow V_{meas} = 1 + (.37mA)\frac{10K(5.76K)}{10K + 5.76K} = 2.35V$ ✓

$R = 16K \rightarrow V_{meas} = 1 + (.37)\frac{16K(5.76K)}{16K + 5.76K} = 2.57V$ ✓

$T = 10^\circ C$: $R = 10K \rightarrow V_{meas} = 1 + (.37)\dfrac{10K(28.51K)}{10K + 28.51K} = \underline{\underline{3.74V}}$ ✓

(extra sheet for work)

$R = 16K \rightarrow V_{meas} = 1 + (.37)\dfrac{16K(28.51K)}{16K + 28.51K} = \underline{\underline{4.79V}}$ X (TOO HIGH)

+9

Eliminate $R = 16K$

☆ Check sensitivity with $R = 10K$

$$\Delta T_{min} = \frac{\Delta V_{min}}{\frac{\partial V_{meas}}{\partial T}} = \frac{\Delta V_{min}}{\frac{\partial V_{meas}}{\partial R_T}\alpha R_T}$$

$$\Delta V_{ADC} = \frac{5-0V}{2^{12}-1} = 1.22mV \quad (\Delta V_{min})$$

$$V_N = 0.5mV$$

$$\frac{\partial V_{meas}}{\partial R_T} = \frac{\partial}{\partial R_T}\left[\frac{V_{ref}}{R_1}R\left(\frac{R_T}{R+R_T}\right)\right] = \frac{V_{ref}R}{R_1}\left[\frac{1}{R+R_T} - \frac{R_T}{(R+R_T)^2}\right]$$

$$= \frac{V_{ref}}{R_1}R\left[\frac{R+\cancel{R_T}}{(R+R_T)^2} - \frac{\cancel{R_T}}{(R+R_T)^2}\right] = \frac{V_{ref}}{R_1}\frac{R^2}{(R+R_T)^2}$$

$$T = 50^\circ C: \frac{\partial V_{meas}}{\partial T} = \frac{1V}{2.7K}\frac{(10K)^2}{(10K + 5.76K)^2}\left(-.0358\tfrac{1}{^\circ C}\right)(5.76K) = -.0308\, V/^\circ C$$

$$|\Delta T_{min}| = \frac{1.22\times 10^{-3}V}{|-.0308\, V/^\circ C|} = 0.0396^\circ C = \boxed{.04^\circ C} < .05^\circ C$$ ✓

+8

⇨ R = 10K works!

Problem #4: ECG Amplifier (25 pts)

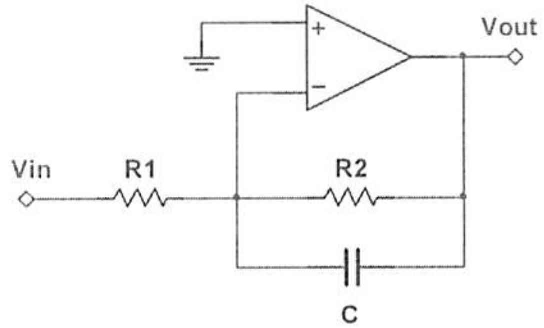
You are asked to design a single-lead ECG system to monitor a person exercising on a stationary bike. Suppose the ECG input is a PQRST waveform with a 0.4 mV amplitude R-wave. The patient's heart rate is 150 beats per minute. Mismatches in the skin-electrode interface produce a constant 10 mV voltage difference between the two electrodes. The person's motion causes a severe common mode voltage resembling a 1 Hz sine wave with a peak-to-peak amplitude of 2 V_{PP}. The instrumentation amplifier has a differential gain A_d = 30 and CMRR = 75 dB.

a) Compute and sketch the instrumentation amplifier output over a 2 second interval. Label important features!

b) The op amp circuit shown to the right combines a low pass filter with an inverting amplifier. Use the Golden Rules to show that:

$$\frac{V_{out}}{V_{in}} = -G_O \frac{1}{1+jf/f_C}$$

NOTE: Clearly define G_O and f_C (e.g. in terms of R1, R2, and/or C)!



c) Let us assume that the circuit from Part (b) passes a signal frequency when $|V_{OUT}/V_{IN}| > 0.9G_O$. Suppose R1 = 1 kohm and R2 = 100 kohm. Assume the ECG signal has frequency content that spans from 1 to 30 Hz. What is the appropriate value for C in order to suppress noise? Choose a standard 10% capacitor value (see table on last page). Show all work!

(a) $V_{OUT} = A_d \Delta V + A_{CM} V_{CM} + 0$ ← $\sin 2\pi f_o t$

+9

30 → A_d; $\Delta V = \left[\underset{PQRST}{.4\ mV} + 10\ mV \right]$

$CMRR = 20\log_{10} \frac{A_d}{A_{CM}}$

$\frac{A_d}{A_{CM}} = 10^{CMRR/20} \Rightarrow A_{CM} = \frac{30}{10^{75/20}}$

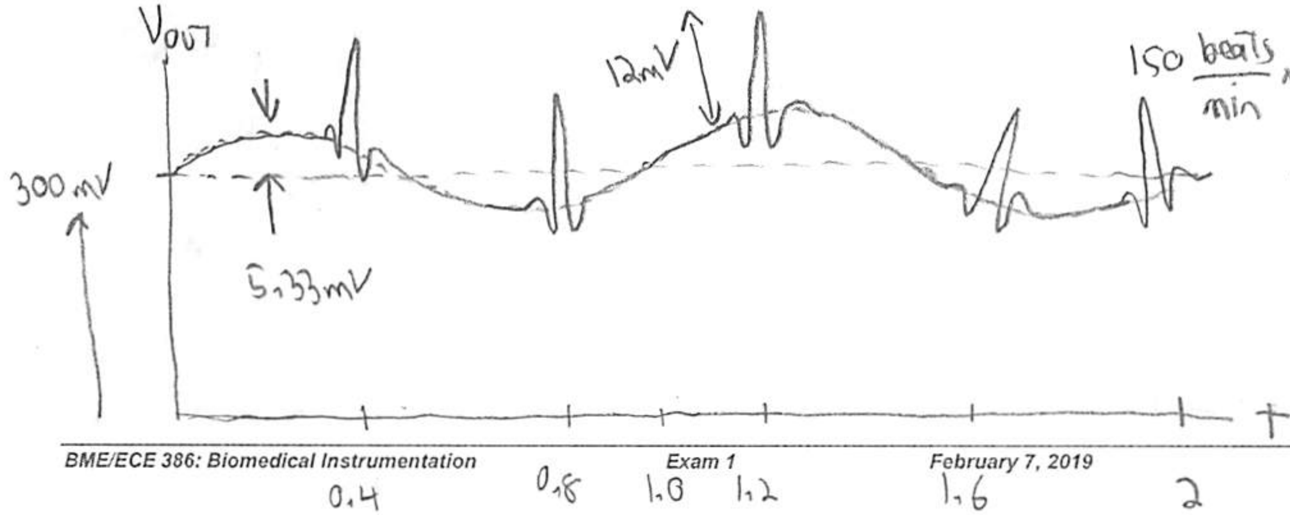
$= 5.33 \times 10^{-3}$

$= \underset{PQRST}{12\ mV} + 300\ mV + (5.33\ mV)\sin 2\pi f_o t$

← 1 Hz (slow!)

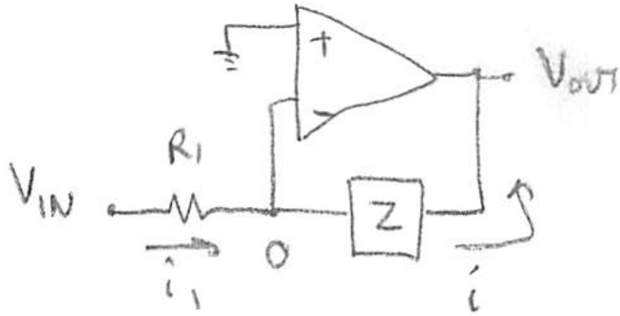
$150 \frac{beats}{min} \times \frac{min}{60 s} = 2.5 \frac{beats}{sec}$

→ 5 beats in 2 sec!



(extra sheet for work) +8

(b) $V_+ = V_- = 0$



KCL at V_-: $i_1 = i$

$$\frac{V_{IN}-0}{R_1} = \frac{0-V_{OUT}}{Z}$$

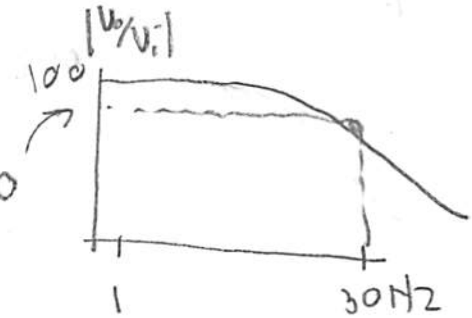
$$Z = \frac{R_2 \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1+j\omega R_2 C}$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{Z}{R_1} = \boxed{-G_0 \frac{1}{1+j f/f_c}}$$

$$= \frac{R_2}{1+j f \cdot \underbrace{2\pi R_2 C}_{1/f_c}}$$

where $G_0 = \frac{R_2}{R_1}$ and $f_c = \frac{1}{2\pi R_2 C}$

(c) $\frac{V_{OUT}}{V_{IN}} = -100 \frac{1}{1+j f/f_c}$



we want

$$\left|\frac{V_o}{V_{in}}\right| \geq 90 \ @ \ f = 30\text{Hz}$$

+8

$$\frac{100}{\sqrt{1+(30/f_c)^2}} \geq 90 \rightarrow \left(\frac{100}{90}\right)^2 \geq 1+(30/f_c)^2$$

$$0.2346 \geq (30/f_c)^2 \rightarrow 0.484 \geq \frac{30}{f_c} = 30 \cdot 2\pi R_2 C$$

$$C \leq \frac{0.484}{30 \cdot 2\pi \cdot 100 \times 10^3 \Omega} = 2.57 \times 10^{-8} \text{F}$$

$$= 25.7 \text{nF}$$

important!

choose $\boxed{C = 22\text{nF}}$