HW 3 – ECG & Op Amps
(9 problems for 100 pts)

OP AMPS

- PROBLEM 1: The op amp circuit in Fig. 1 is an inverting amplifier.
  a) Use the Golden Rules to show that the amplifier gain is V_{OUT}/V_{IN} = -R2/R1.
  b) Let R1 = 15 kohm and R2 = 75 kohm. Given the input waveform shown in Fig. 1, sketch the output. Label important features (e.g. amplitude).

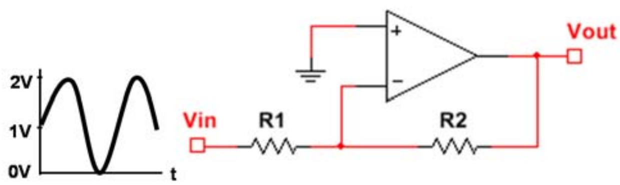

Fig. 1: Op amp configured as an inverting amplifier.

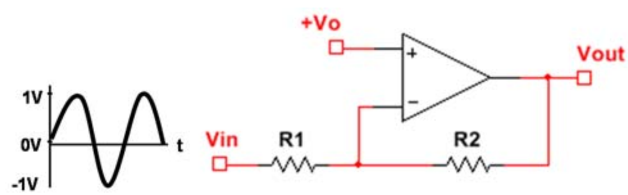

Fig. 2: This is called a level-shifting circuit.

- PROBLEM 2: The op amp circuit in Fig. 2 is called a level-shifting circuit. The input signal is inverted and shifted by an offset voltage.
  a) Use the Golden Rules to derive an expression for V_{OUT}.
  b) Let R1 = 10 kohm and R2 = 75 kohm, and V_O = 0.5V. Given the input waveform shown in Fig. 2, sketch the output. Label important features!

ECG Circuitry

In Lab3, the ECG amplifier used an instrumentation amplifier, a high-pass filter, and then a combined amplifier + low-pass filter. An advantage of this setup is simple circuitry. A disadvantage is that the low-pass filter is only a first-order filter. So what? A first-order low-pass filter has a fairly gradual frequency response. This means our precious ECG signal still contains some high frequency noise. A 2^{nd} order filter provides better performance.

- PROBLEM 3: Consider the 2^{nd} order Butterworth low-pass filter (see Fig. 3). Use the Golden Rules to derive:

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{1+j2\omega RC-2\omega^2R^2C^2}$$

Hint #1: Since the op amp is basically a buffer, $V_+ = V_- = V_{OUT}$.
Hint #2: The current in the "right" resistor R is equal to the current in C. This lets you solve for the voltage V_A at "Node A" in terms of V_{OUT}.
Hint #3: Use KCL at "Node A" to obtain V_{OUT}/V_{IN}.

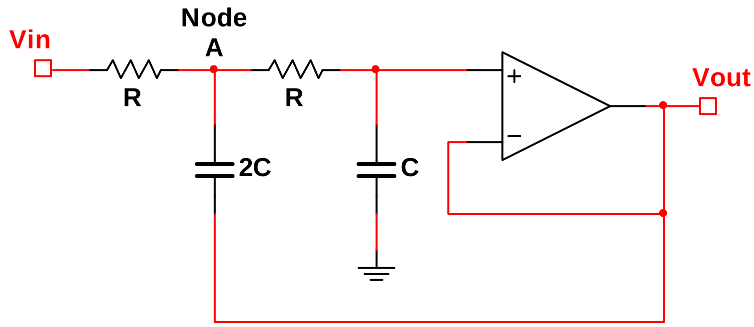

Fig. 3: 2nd order low-pass Butterworth filter.

- PROBLEM 4: Almost there! Starting with your answer to Problem 3, show that:

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+(f/f_C)^4}} \qquad \text{where } f_C = \frac{1}{2\sqrt{2}\pi RC}$$

Hint: For a complex number Y = 1/(a+jb), the magnitude is |Y| = 1/sqrt($a^2 + b^2$).

- PROBLEM 5: The magnitude of a filter's frequency response is often expressed in decibels, where $dB = 20log_{10}|V_{OUT}/V_{IN}|$. This is often called the filter gain. For example, $|V_{OUT}/V_{IN}| = \frac{1}{2}$ corresponds to a filter gain of -6 dB. What is the filter gain at $f = f_C$ for the 2^{nd} order Butterworth filter?

- PROBLEM 6: Let us define f_{STOP} as the frequency where the filter gain is -20 dB. We can then say the low-pass filter blocks all frequencies above f_{STOP}. Compute f_{STOP} in terms of f_C.

  NOTE: You should find that $f_{STOP} = 3.15f_C$. Meanwhile, a 1^{st} order filter has $f_{STOP} = 9.95f_C$. So what? A lower f_{STOP} means a sharper cut-off, resulting in more noise rejection and a cleaner signal!

Suppose an ECG input is a PQRST waveform with a 300 μV amplitude R-wave. The person has a heart rate of 80 beats per minute. This desired signal is superimposed with a constant 50 mV differential offset voltage (e.g. due to skin-electrode potential mismatch). The 60 Hz power line introduces a severe common mode voltage with a peak-to-peak amplitude of 4 V.

- PROBLEM 7: Assuming a differential gain A_d = 25 and CMRR = 90 dB, compute and sketch the instrumentation amplifier output over a 3 second interval. Label important features (e.g. time between beats, amplitude of R-wave, amplitude of power line interference, DC offset).

The output of the instrumentation amplifier has a large DC offset that can be removed with a simple RC high-pass filter. This filter should be designed to have minimal impact on the PQRST waveform. We must therefore know the temporal properties of the PQRST waveform! The P-wave and QRS-complex are each about 80 ms in duration, while the T-wave is about 160 ms in duration. In PreLab3, you should have found that frequencies above f_{PASS} = $2.07f_C$ are mostly unaffected by the high-pass filter.

- PROBLEM 8: Suppose you had to decide between wither f_C = 1 Hz and 10 Hz. Explain why one value is a reasonable choice while the other value is a poor choice.

  Hint: You do not want the high-pass filter to affect the frequency content of the P, QRS, and T portions of the ECG waveform. The frequency bandwidth of each portion is roughly equal to 1/(pulse duration).

The very high input impedance of an instrumentation amplifier makes it especially suitable for applications involving sensors with high output impedance. To illustrate this point, let us consider the thermistor circuit shown in Fig. 4. Suppose the thermistor has R_T = 5 kohm and the other three resistors are 10.0 kohm.

- PROBLEM 9: Instead of an instrumentation amplifier, suppose we foolishly use a single op-amp differential amplifier as shown in Fig. 4. This amplifier is supposed to have a differential gain of $A_d = R_2/R_1 = 3$. Using a similar procedure to the example in the Lecture 7 notes, show that V_{MEAS} is about 23% lower than the result using an instrumentation amplifier with a gain A_d = 3. That's a pretty significant error!

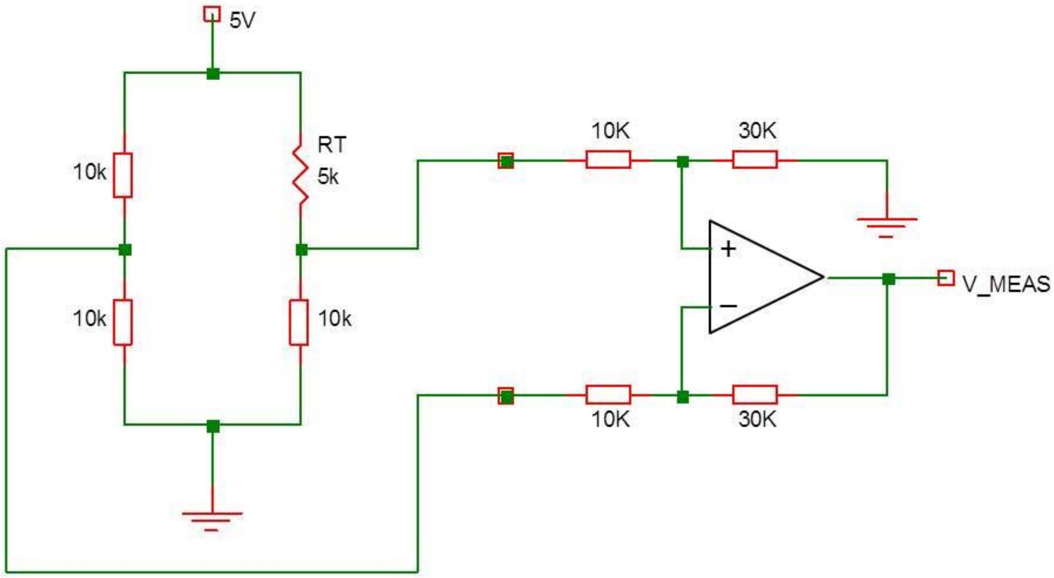

Fig. 4: Thermistor circuit with single op-amp differential amplifier.