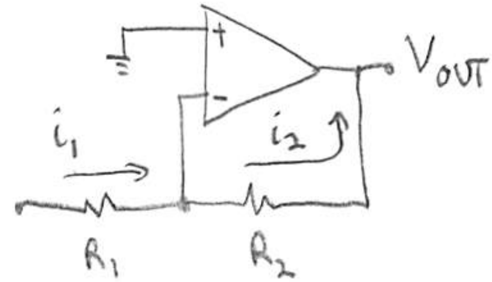
BME 386
HW3 Solns

9 problems
Total = 100 pts

1) (a) $V_- = V_+ = \underline{\underline{0}}$



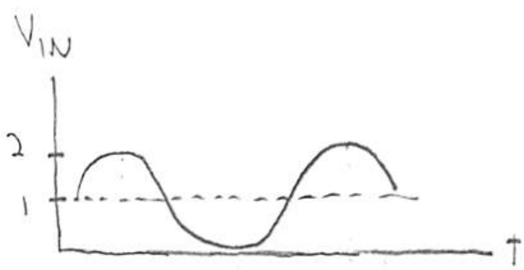
KCL at node V_-:

$$i_1 = i_2 + 0$$

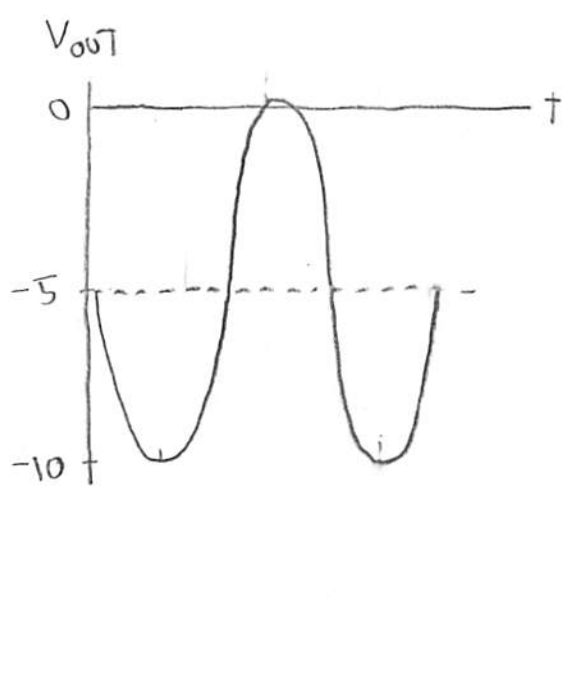
$$\frac{V_{IN} - \cancel{V_-}^{0}}{R_1} = \frac{\cancel{V_-}^{0} - V_{OUT}}{R_2} \implies \frac{V_{IN}}{R_1} = -\frac{V_{OUT}}{R_2}$$

+6 $\boxed{\frac{V_{OUT}}{V_{IN}} = -\frac{R_2}{R_1}}$

(b) $\frac{V_{OUT}}{V_{IN}} = -\frac{75K}{15K} = -5$ ← invert!

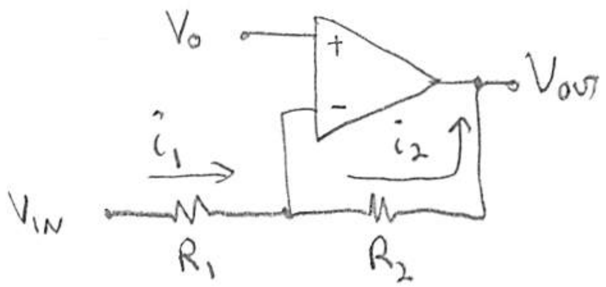


+6



2) ⓐ +6

$V_- = V_+ = V_0$



KCL at node V_-:

$$i_1 = i_2 + 0$$

$$\frac{V_{IN} - V_-}{R_1} = \frac{V_- - V_{OUT}}{R_2} \Rightarrow \frac{V_{IN} - V_0}{R_1} = \frac{V_0 - V_{OUT}}{R_2}$$

$$\frac{R_2}{R_1} V_{IN} - \frac{R_2}{R_1} V_0 - V_0 = -V_{OUT}$$

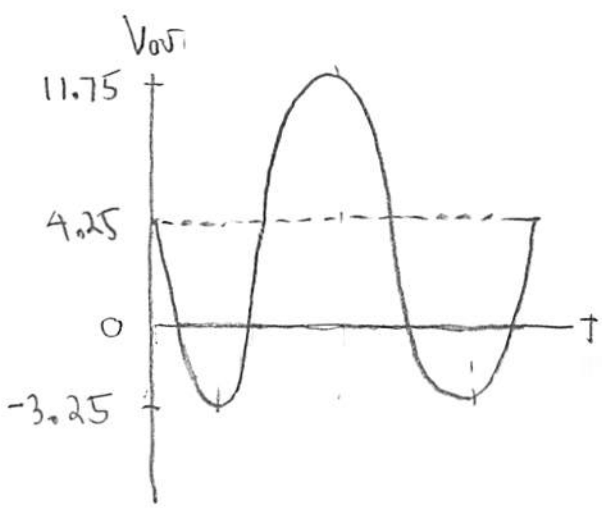
$$\Rightarrow \boxed{V_{OUT} = \left(1 + \frac{R_2}{R_1}\right) V_0 - \frac{R_2}{R_1} V_{IN}}$$

ⓑ $V_{OUT} = \left(1 + \frac{75K}{10K}\right)(0.5V) - \frac{75K}{10K} V_{IN}$

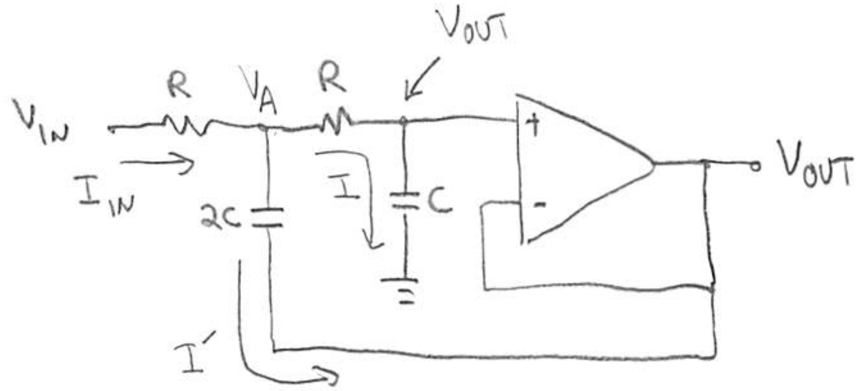
$$= \underbrace{4.25}_{\substack{\text{DC} \\ \text{offset}}} - \overset{}{7.5}\, V_{IN}$$

↑ invert!

+6



3) +12



a) Rule 1: $V_+ = V_- = V_{OUT}$

$$I = \frac{V_A - V_{OUT}}{R} = \frac{V_{OUT}}{Z}, \qquad Z = \frac{1}{j\omega C}$$

$$\rightarrow V_A = \left(\frac{R}{Z} + 1\right) V_{OUT} \quad (1)$$

KCL at node A:

$$I_{IN} = I + I'$$

$$\frac{V_{IN} - V_A}{R} = \frac{V_{OUT}}{Z} + \frac{V_A - V_{OUT}}{\frac{1}{2} Z} \qquad \frac{1}{j\omega 2C} = \frac{1}{2} Z$$

$$V_{IN}\frac{Z}{R} = \frac{Z}{R} V_A + \cancel{V_{OUT}} + 2V_A - \cancel{2}V_{OUT}$$

$$= \frac{Z}{R}\left(\frac{R}{Z} + 1\right) V_{OUT} + 2\left(\frac{R}{Z} + 1\right) V_{OUT} - V_{OUT}$$

$$= \left(\cancel{1} + \frac{Z}{R} + 2\frac{R}{Z} + 2 - \cancel{1}\right) V_{OUT}$$

$$\frac{V_{OUT}}{V_{IN}} = \frac{Z/R}{\frac{Z}{R} + 2 + 2R/Z} = \frac{1}{1 + 2R/Z + 2R^2/Z^2}$$

Let $Z = \frac{1}{j\omega C} \rightarrow \frac{1}{Z} = j\omega C$, $\frac{1}{Z^2} = -\omega^2 C^2$

$$\rightarrow \boxed{\frac{V_{OUT}}{V_{IN}} = \frac{1}{1 + 2j\omega RC - 2\omega^2 R^2 C^2}}$$

4)

+12

$$\frac{V_{OUT}}{V_{IN}} = \frac{1}{(1 - 2\omega^2 R^2 C^2) + j\,2\omega RC}$$

$$\left| \frac{V_{OUT}}{V_{IN}} \right| = \frac{1}{\sqrt{(1 - 2\omega^2 R^2 C^2)^2 + (2\omega RC)^2}}$$

$$= \frac{1}{\sqrt{1 \cancel{- 4\omega^2 R^2 C^2} + 4\omega^4 R^4 C^4 \cancel{+ 4\omega^2 R^2 C^2}}}$$

$$= \frac{1}{\sqrt{1 + 4\underset{\substack{\uparrow \\ 2\pi f}}{\omega^4} R^4 C^4}} = \frac{1}{\sqrt{1 + \underbrace{64\pi^4 R^4 C^4}_{1/f_c^4} f^4}}$$

Let $f_c = \frac{1}{64^{1/4}\pi RC}$ $\qquad 64^{1/4} = (8^2)^{1/4} = \sqrt{8} = 2\sqrt{2}$

$$\rightarrow f_c = \frac{1}{2\sqrt{2}\,\pi RC}$$

SO, $\boxed{\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+(f/f_c)^4}}}$ $\Leftarrow$ $f_c = \frac{1}{2\sqrt{2}\,\pi RC}$

5) When $f = f_c \rightarrow \left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{1}{\sqrt{1+(1)^4}} = \frac{1}{\sqrt{2}}$

+8

$$20\log_{10}\frac{1}{\sqrt{2}} = \boxed{-3\text{ dB}}$$

6) $20\log_{10}\left|\frac{V_{OUT}}{V_{IN}}\right| = -20\text{ dB}$

+8

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = 10^{-20/20} = 0.1 = \frac{1}{\sqrt{1+(f_{STOP}/f_c)^4}}$$

$$.01 = \frac{1}{1+(f_{STOP}/f_c)^4}$$

$$\frac{f_{STOP}}{f_c} = (100-1)^{1/4}$$

$$f_{STOP} = 99^{1/4} f_c = \boxed{3.15 f_c}$$

7) Instrumentation amplifier output is: $V_{OUT} = A_d \Delta V + A_{CM} V_{CM}$

+12

- $A_d = 25$
- $\Delta V = 50\,mV + (300\,\mu V \text{ R-wave})$

$$\left.\begin{array}{l} A_d = 25 \\ \Delta V = 50\,mV + (300\,\mu V \text{ R-wave}) \end{array}\right\} A_d \Delta V = 1.25V + (7.5\,mV \text{ R-wave})$$

- A_{CM}?: $CMRR = 20\log_{10}\frac{A_d}{A_{CM}} \rightarrow \frac{A_d}{A_{CM}} = 10^{CMRR/20}$

$$A_{CM} = A_d\, 10^{-CMRR/20}$$
$$= 25 \times 10^{-90/20} = \underline{\underline{7.9 \times 10^{-4}}}$$

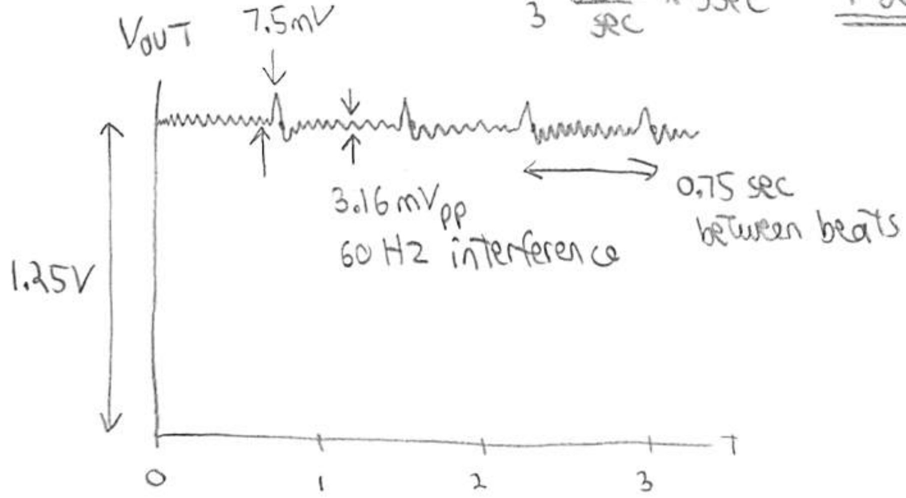
$V_{CM} = 2\sin 2\pi f_0 T$

f_0: 60 Hz $\Rightarrow A_{CM} V_{CM} = .00158 \sin 2\pi f_0 T$

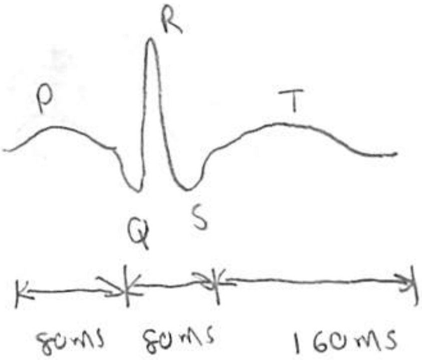
So, $\boxed{V_{OUT} = 1.25 + (.0075V \text{ R-wave}) + .00158 \sin 2\pi f_0 t}$

80 beats per minute $\rightarrow \frac{80 \text{ beats}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ sec}} = \frac{4}{3}\frac{\text{beats}}{\text{sec}} \rightarrow$ 0.75 sec between beats

$$\frac{4}{3}\frac{\text{beats}}{\text{sec}} \times 3\,\text{sec} = \underline{\underline{4 \text{ beats}}}$$



8) PQRST waveform:



+12

The bandwidth of a pulse is roughly given by:

$$BW \sim \frac{1}{\text{duration}}$$

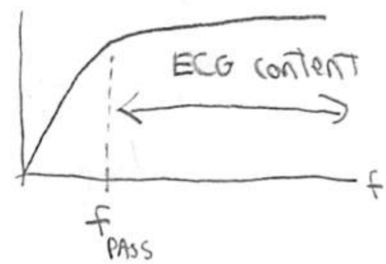
P-wave: $BW \sim \frac{1}{.08\,s} = 12.5\text{ Hz}$

QRS : $BW \sim \frac{1}{.08\,s} = 12.5\text{ Hz}$

T-wave : $BW \sim \frac{1}{.16\,s} = 6.25\text{ Hz}$

we want $f_{PASS} <$ Smallest BW in PQRST waveform.

$< 6.25\text{ Hz}$

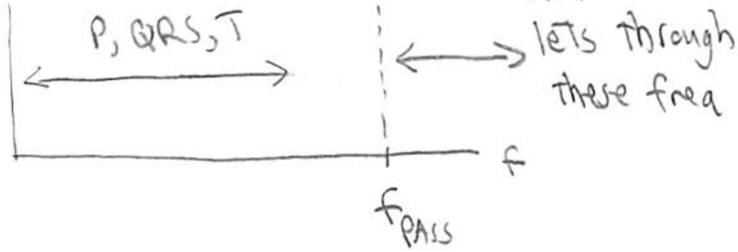


- $f_c = 1\text{ Hz}$ is a good choice because $f_{PASS} = 2.07\text{ Hz}$ is well below 6.25 Hz

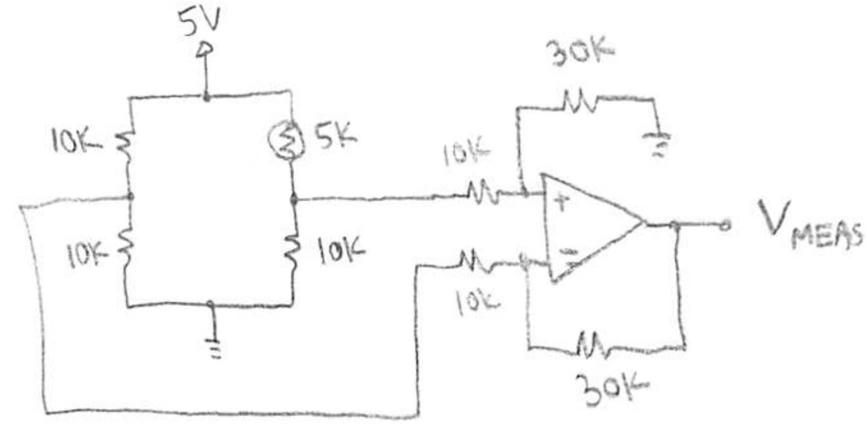
- $f_c = 10\text{ Hz}$ is a poor choice because $f_{PASS} = 20.7\text{ Hz}$ is above the bandwidth of P, QRS, and T portions!

$\Downarrow$

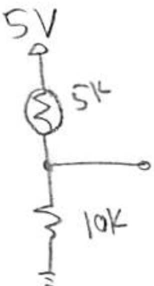
ECG waveform is significantly affected by HPF.



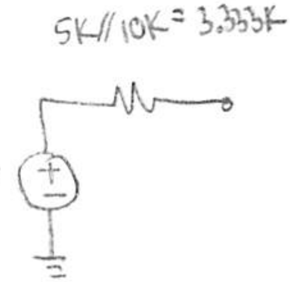
9) (+12)

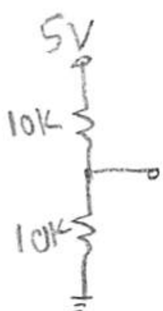


Convert bridge to Thevenin equivalent:

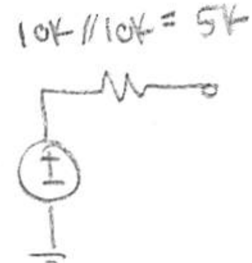


$$= \quad 5 \times \frac{10K}{15K} = 3.333V$$

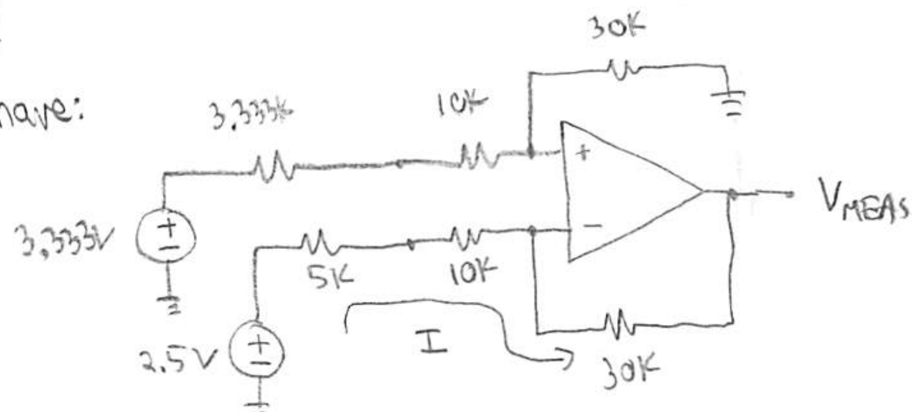




$$= \quad 5 \times \frac{10K}{20K} = 2.5V$$



we now have:



$$V_+ = (3.333V)\ \frac{30K}{3.333K + 10K + 30K} = \underline{\underline{2.31V}} = V_-$$

$$I = \frac{2.5V - 2.31V}{5K + 10K} = \underline{\underline{.01267mA}}$$

So, $V_{MEAS} = V_- - I \times 30K = 2.31V - (.01267mA)(30k) = \boxed{1.93V}$

An instrumentation amplifier with $A_d = 3$ would produce:

$$V_{MEAS} = 3 \times (3.333V - 2.5V) = \underline{\underline{2.5V}}$$

$$\text{Error} = \frac{1.93 - 2.5}{2.5} = -0.228 = \boxed{-22.8\%}$$