PreLab 3: ECG Measurement System
(12 problems for 40 pts)

The overall gain of the ECG system should be about 2000. However, not all of this gain should be produced by the instrumentation amplifier. The reason is that a differential DC voltage of up to 200 mV can exist at the input of the amplifier. An excessively high gain A_d will amplify this offset and saturate the amplifier! ☹

- PROBLEM 1: Let your instrumentation amplifier be powered by +/-9 V. Assume the maximum signal output is 1V less than the power supply. What is the maximum usable gain for the instrumentation amplifier?

  Note: For Lab3, we will be a little conservative and use a gain $A_d = 20$ (produced by $R_G = 2.61$ kohm).

The instrumentation amplifier output consists of the desired ECG signal with a DC offset. In order to block this annoying DC offset, ECG systems should have a high pass filter that rejects frequencies lower than ≈ 0.1 Hz.

- PROBLEM 2: Show that the transfer function of the RC high pass filter is given by:

$$\frac{V_{out}}{V_{in}} = \frac{jf/f_C}{1 + jf/f_C}$$

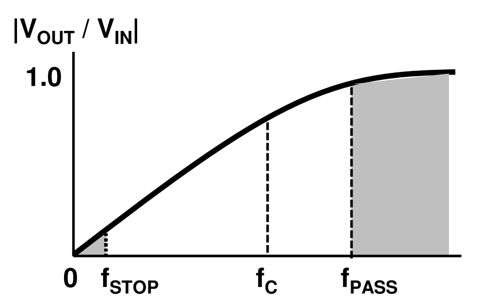
where $f_C = 1/(2\pi RC)$.

  Hint: Remember that the impedance of a capacitor is $Z = 1/j\omega C$.

For Problems 3 to 6, assume R = 1 Mohm and C = 0.1 µF.

- PROBLEM 3: f_C is called the "corner frequency". Compute the value of f_C as well as the magnitude $|V_{OUT}/V_{IN}|$ at $f = f_C$. Hint: You should get $|V_{OUT}/V_{IN}| = 1/\sqrt{2} = 0.707$ at $f_C = 1.6$ Hz.

Although f_C is called the "corner frequency", our simple high pass filter does not abruptly cut off frequencies below f_C. Let us define f_{STOP} as the frequency below which the high-pass filter has $|V_{OUT}/V_{IN}| \leq 0.1$. This means the high pass filter rejects frequencies in the range $0 < f < f_{STOP}$.



- PROBLEM 4: What is f_{STOP} for your high pass filter? Hint: You should get something like 0.16 Hz.

- PROBLEM 5: Let us define f_{PASS} as the frequency where $|V_{OUT}/V_{IN}| = 0.9$. This means the high pass filter lets through frequencies above f_{PASS}. What is f_{PASS} for your high pass filter? Hint: You should get around 3.3 Hz.

- PROBLEM 6: Sketch $|V_{OUT}/V_{IN}|$ from 0 to 10 Hz. Label your axes as well as f_{STOP}, f_C, and f_{PASS}. Also label the value of $|V_{OUT}/V_{IN}|$ at DC, f_{STOP}, f_C, and f_{PASS}.

After the ECG signal is high pass filtered, it must be amplified and low-pass filtered. As explained in lecture, we can very happily do both with a non-inverting amplifier, where a capacitor C_2 is added in parallel with resistor R_2.

- PROBLEM 7: Use the Golden Rules to show that the amplifier gain $G = V_{OUT}/V_{IN}$ is given by:

$$G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}\frac{1}{1 + jf/f_C} \qquad \text{where } f_C = 1/(2\pi R_2 C_2).$$

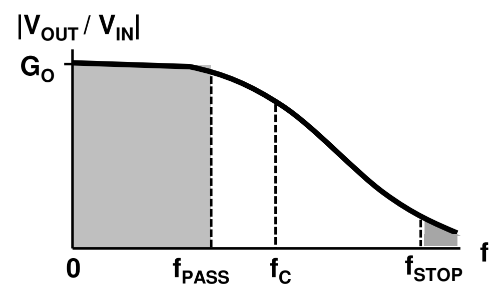
Hint: The math is easier if you first replace the parallel R_2 and C_2 with an effective impedance Z_2. Then use the Golden Rules to determine V_{OUT}/V_{IN}, and finally plug in the full expression for Z_2.

For Problems 8 – 12, assume R_1 = 1 kohm, R_2 = 100 kohm, and C_2 = 0.1 µF.

- PROBLEM 8: Let G_0 be the amplifier gain at DC. Compute the value of G_0.

- PROBLEM 9: Compute the value of the corner frequency f_C and the magnitude of the gain |G| at $f = f_C$. Hint: You should get |G| = 71.4 at f_C = 15.9 Hz.

- PROBLEM 10: Let us define f_{PASS} as the frequency where $|G| = 0.9G_0$. This means that the amplifier has high gain for $0 < f < f_{PASS}$. Compute the value of f_{PASS} for your amplifier. Hint: You should get f_{PASS} = 7.7 Hz.



Hint #1: The math is a little easier if your first plug in the values for R1 and R2.

Hint #2: The magnitude of the ratio of two complex numbers is: $\left|\frac{a+jb}{c+jd}\right| = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}}$

In order to suppress noise, ECG systems should block frequencies higher than about 150 Hz. As explained in lecture, the purpose of the capacitor C_2 is to reduce the amplifier gain at high frequencies. Although f_C is often called the "corner frequency", the amplifier gain does not abruptly plummet at frequencies above f_C.

- PROBLEM 11: Let us define f_{STOP} as the frequency where $|G| = 0.1G_0$. This means the amplifier has very low gain for $f > f_{STOP}$. Compute f_{STOP} for your amplifier. Hint: You should get f_{STOP} = 159 Hz.

- PROBLEM 12: Sketch |G| from 0 to 300 Hz. Label your axes as well as f_{PASS}, f_C and f_{STOP}. Also label the value of |G| at DC, f_{PASS}, f_C, and f_{STOP}.

(End of PreLab 3)