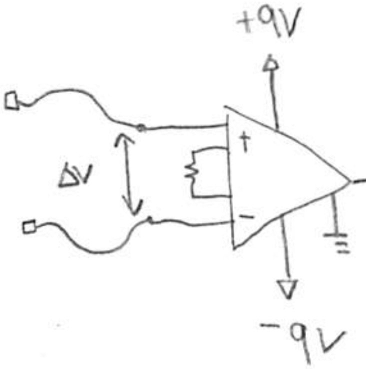
BME 386

PreLab 3 Solns

Total = 30 pts

① +2

$$\Delta V = \overbrace{(PQRST)}^{\lesssim 1\,mV} + \underset{\substack{\uparrow \\ \text{Max differential} \\ \text{offset}}}{0.2V}$$



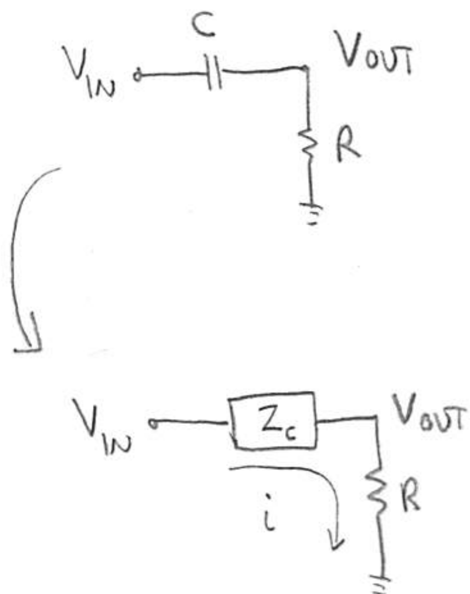
$V_{OUT} = A_d \Delta V$

Max $\Delta V \sim 0.2V \longrightarrow$ Max $V_{OUT} < 8V$

$A_d \times 0.2V < 8V \Rightarrow \boxed{A_d < 40}$

② +2

$$V_{OUT} = iR = \frac{V_{IN}}{Z_c + R} R$$



$$\frac{V_{OUT}}{V_{IN}} = \frac{R}{Z_c + R}$$

$$= \frac{R}{\frac{1}{j\omega C} + R} = \frac{j\omega RC}{1 + j\omega RC}$$

using $\omega = 2\pi f$

where $Z_c = \frac{1}{j\omega C}$

$$\rightarrow \frac{V_{OUT}}{V_{IN}} = \frac{jf \cdot 2\pi RC}{1 + jf \cdot 2\pi RC}$$

let $f_c = \frac{1}{2\pi RC} \Longrightarrow \boxed{\frac{V_{OUT}}{V_{IN}} = \frac{jf/f_c}{1 + j\,f/f_c}}$

(3) $f_c = \dfrac{1}{2\pi(10^6\Omega)(0.1\times10^{-6}F)} = \boxed{1.6\text{ Hz}}$ (+3)

$$\frac{V_{OUT}}{V_{IN}} = \frac{j f/f_c}{1+j f/f_c} \xrightarrow{\text{Let } b = f/f_c} \frac{V_{OUT}}{V_{IN}} = \frac{jb}{1+jb}$$

Many ways to compute $\left|\dfrac{V_{OUT}}{V_{IN}}\right|$ ...

$\boxed{\text{Method 1}}$ Use $\left|\dfrac{a+jb}{c+jd}\right| = \dfrac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \Rightarrow \left|\dfrac{V_{OUT}}{V_{IN}}\right| = \dfrac{b}{\sqrt{1+b^2}}$ (1)

when $f=f_c \rightarrow b=1 \Rightarrow \boxed{\left|\dfrac{V_{OUT}}{V_{IN}}\right| = \dfrac{1}{\sqrt{2}} = 0.707}$

$\boxed{\text{Method 2}}$ Make denominator real:

$$\frac{V_{OUT}}{V_{IN}} = \frac{jb}{1+jb} \times \frac{1-jb}{1-jb} = \frac{b^2+jb}{1+b^2} = \frac{b}{1+b^2}(b+j)$$

Then take magnitude:

$$\left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{b}{1+b^2}|b+j| = \frac{b}{1+b^2}\sqrt{b^2+1} = \frac{b}{\sqrt{1+b^2}} \leftarrow \text{same as (1)}$$

$\boxed{\text{Method 3}}$ Re-write both numerator and denominator as phasors:

$$\frac{V_{OUT}}{V_{IN}} = \frac{jb}{1+jb} = \frac{b e^{j\pi/2}}{\sqrt{1+b^2}\, e^{j\tan^{-1}(b)}} = \frac{b}{\sqrt{1+b^2}} e^{j(\pi/2 - \tan^{-1} b)}$$

$$\rightarrow \left|\frac{V_{OUT}}{V_{IN}}\right| = \frac{b}{\sqrt{1+b^2}} \leftarrow \text{same as (1)}$$

(4) When $f = f_{STOP}$, $\left|\frac{V_{OUT}}{V_{IN}}\right| = 0.1$ (+2)

Let $b_s = \frac{f_{STOP}}{f_c} \rightarrow \frac{b_s}{\sqrt{1+b_s^2}} = 0.1 = \frac{1}{10}$

$$100\, b_s^2 = 1 + b_s^2 \Rightarrow b_s = \sqrt{\frac{1}{99}}$$

1.6 Hz

$$f_{STOP} = \sqrt{\frac{1}{99}}\, f_c = \boxed{0.16 \text{ Hz}}$$

(5) When $f = f_{PASS}$, $\left|\frac{V_{OUT}}{V_{IN}}\right| = 0.9$

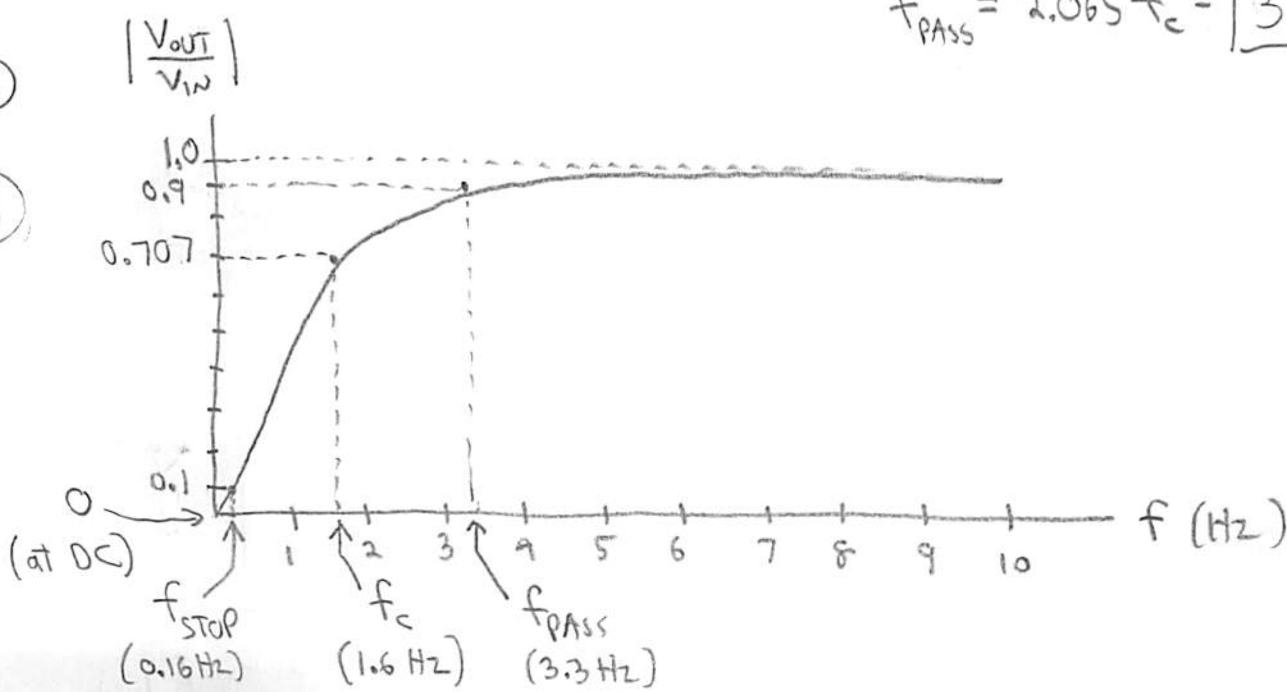
Let $b_p = \frac{f_{PASS}}{f_c} \rightarrow \frac{b_p}{\sqrt{1+b_p^2}} = 0.9$ (+2)

$$b_p^2 = 0.81\,(1 + b_p^2)$$

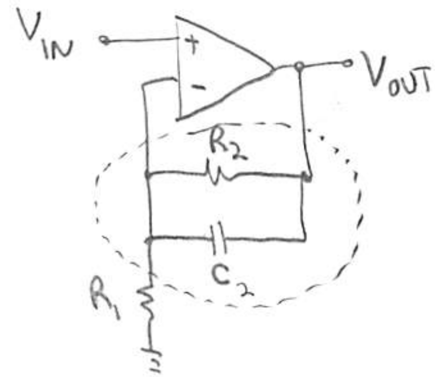
$$0.19\, b_p^2 = 0.81 \Rightarrow b_p = \sqrt{\frac{.81}{.19}} = 2.065$$

$$f_{PASS} = 2.065\, f_c = \boxed{3.3 \text{ Hz}}$$

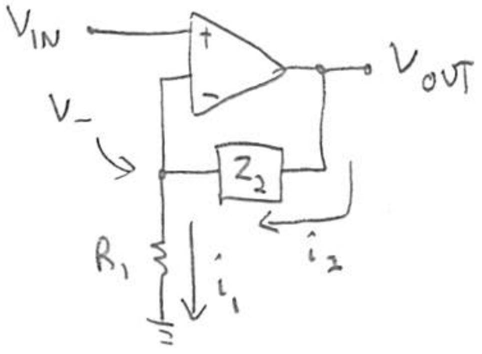
(6) (+3)



(7) +3



$\Rightarrow$



- Rule #1: $V_- = V_+$
  $= V_{IN}$

- KCL at node V_-:

$$i_2 = i_1 + \underbrace{0}_{\text{Rule \#2}}$$

$$\frac{V_{OUT} - V_-}{Z_2} = \frac{V_- - 0}{R_1}$$

$$V_{OUT} = \left(1 + \frac{Z_2}{R_1}\right) V_- = \left(1 + \frac{Z_2}{R_1}\right) V_{IN}$$

$$\Rightarrow \frac{V_{OUT}}{V_{IN}} = 1 + \frac{Z_2}{R_1} = 1 + \frac{R_2}{R_1} \frac{1}{1 + j\omega R_2 C_2}$$

$$= 1 + \frac{R_2}{R_1} \frac{1}{1 + j f \underbrace{2\pi R_2 C_2}_{1/f_c}} \Rightarrow \boxed{\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_2}{R_1} \frac{1}{1 + j f/f_c}}$$

where $f_c = \frac{1}{2\pi R_2 C_2}$

$$Z_2 = R_2 // \frac{1}{j\omega C_2} = \frac{R_2 \frac{1}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} = \frac{R_2}{1 + j\omega R_2 C_2}$$

(8) DC means $f = 0$! $\frac{V_{OUT}}{V_{IN}} = 1 + \frac{R_2}{R_1} = G_0 \Rightarrow G_0 = 1 + \frac{100K}{1K} = \boxed{101}$

+2

(9) $f_c = \frac{1}{2\pi (10^5 \Omega)(0.1 \times 10^{-6} F)} = \boxed{15.9 \text{ Hz}}$

+3

Need expression for $|G|$

$$G = 1 + 100 \frac{1}{1 + j\, f/f_c}$$

Let $b = f/f_c \Rightarrow G = 1 + 100 \frac{1}{1+jb} = \frac{1+jb}{1+jb} + \frac{100}{1+jb} = \frac{101 + jb}{1+jb}$

using $\left| \frac{a+jb}{c+jd} \right| = \frac{\sqrt{a^2+b^2}}{\sqrt{c^2+d^2}} \rightarrow |G| = \frac{\sqrt{101^2 + b^2}}{\sqrt{1+b^2}}$

When $f = f_c \rightarrow b = 1 \Rightarrow |G| = \frac{\sqrt{101^2+1}}{\sqrt{2}} = \boxed{71.4}$

(10) Let $b_p = \frac{f_{PASS}}{f_c} \rightarrow 0.9\, G_0 = \frac{\sqrt{101^2 + b_p^2}}{\sqrt{1 + b_p^2}}$

+2

$$0.81\, G_0^2 (1 + b_p^2) = 101^2 + b_p^2$$

$$0.81 (101)^2 b_p^2 - b_p^2 = 101^2 - 0.81 (101)^2$$

$$8261.81\, b_p^2 = 1938.19 \rightarrow b_p = 0.484$$

15.9 Hz

$$f_{PASS} = 0.484\, f_c$$

$$= \boxed{7.7\ \text{Hz}}$$

(11) Let $b_s = \frac{f_{STOP}}{f_c} \rightarrow 0.1 \times 101 = \frac{\sqrt{101^2 + b_s^2}}{\sqrt{1+b_s^2}}$

+3

$$(10.1)^2 (1 + b_s^2) = 101^2 + b_s^2$$

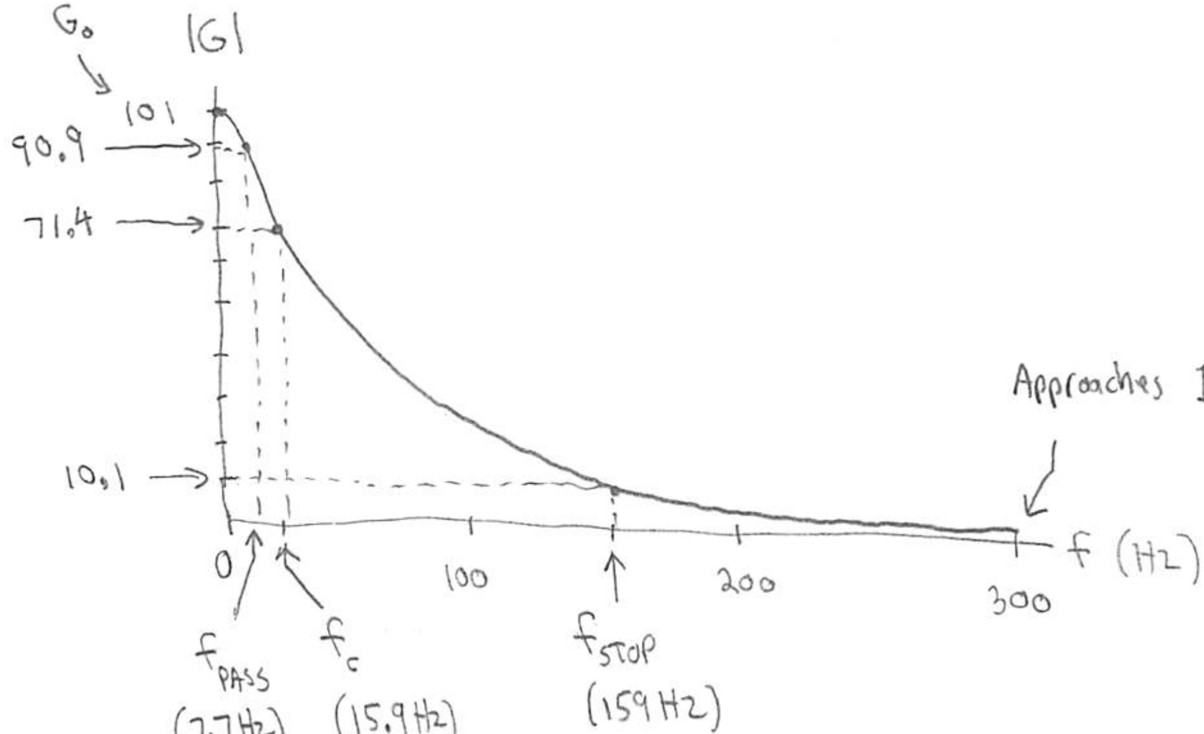
$$102.01\, b_s^2 - b_s^2 = 101^2 - 102.01$$

$$101.01\, b_s^2 = 10098.99$$

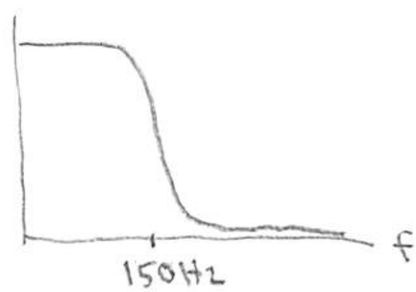
$$b_s = 9.999 \rightarrow f_{STOP} = 9.999 \underset{\substack{\uparrow \\ 15.9\,Hz}}{f_c} = \boxed{159\ Hz}$$

(12)

+3



NOTE: It would be MUCH better to have something like



But this requires a $>4^{th}$ order filter

(more complicated circuit)