Lecture 1: Strain Gauges

0. Course Intro
1. Strain
2. Strain Gauge
3. Strain Measurement

- Course website: minerva.union.edu/bumat
- Reading: Page 1-4 of Strain Gauge Tutorial pdf
- NO LAB THIS WEEK!
- PreLab 1 due at lab session (Jan 14/16)
- HW 1 due next Thu (Jan 16)

- Office Hours: Mon 2-4 PM
  Wed 9-11 AM

You are welcome to swing by at other times, but I may not be available!

or appointment (set up by email)

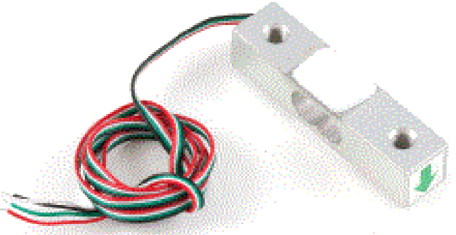
bumat@union.edu

0. Course Intro

★ Course website: minerva.union.edu/bumat

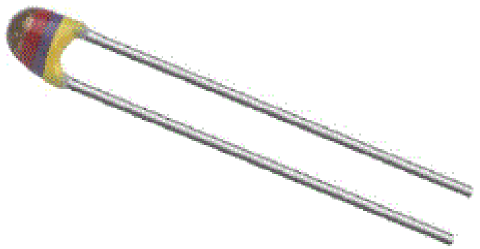
- The lectures and labs for this course will concentrate on:

① Strain sensors (e.g. load cells)

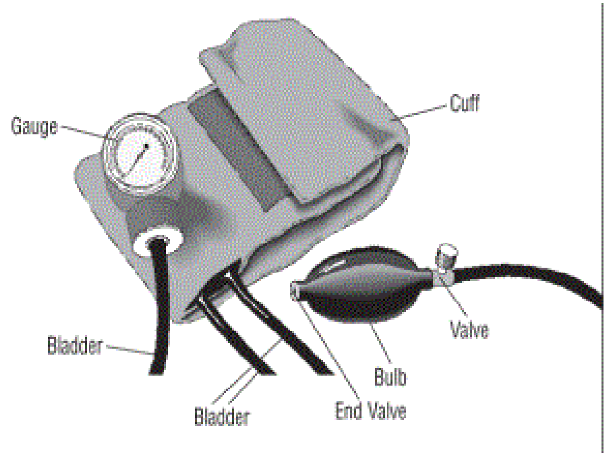


④ optical heart rate monitor

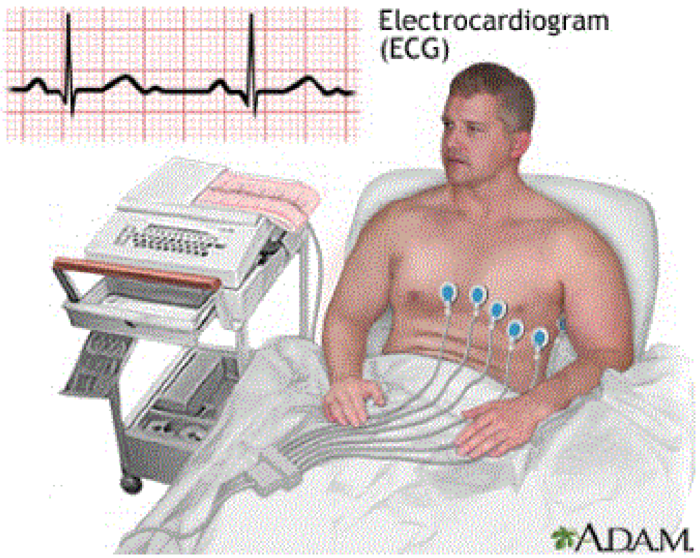
② Temperature (e.g. thermistor)



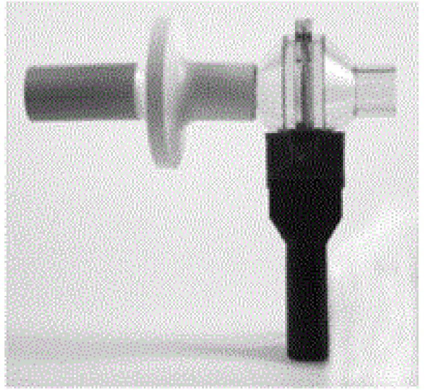
⑤ Blood pressure (e.g. sphygmomanometer)



③ ECG

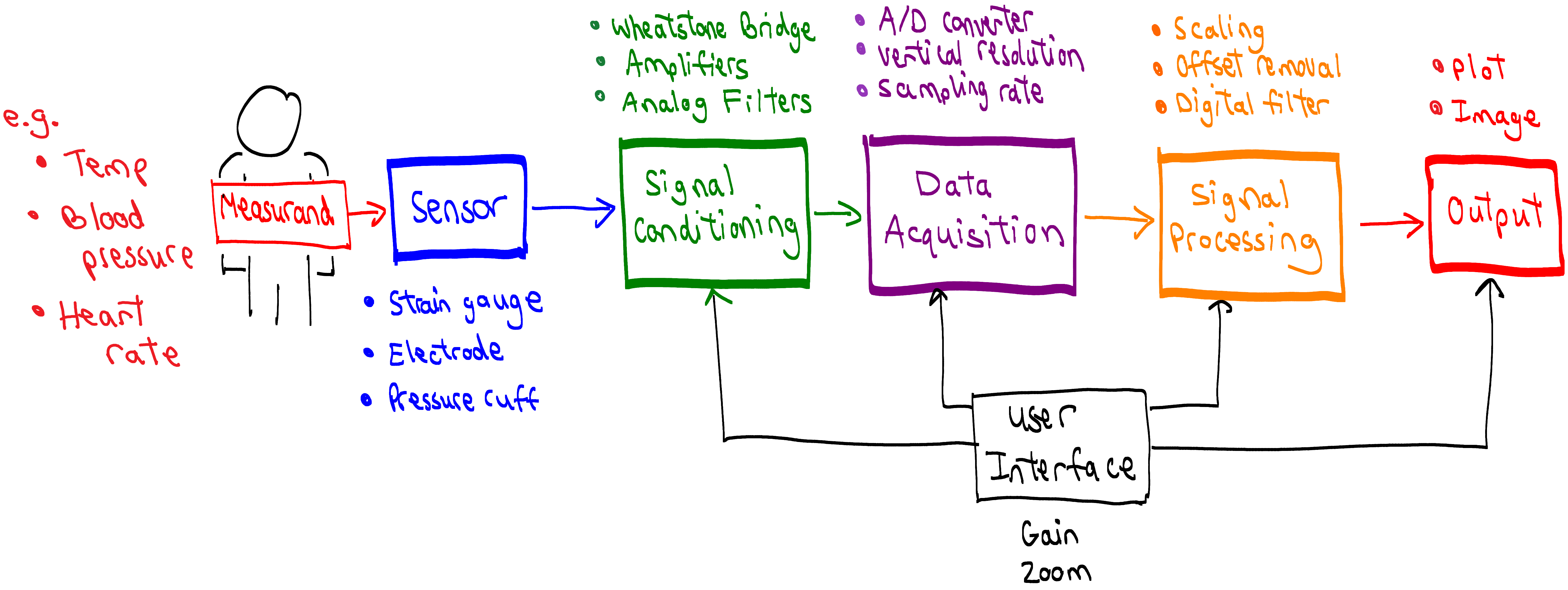


⑥ Respiration (e.g. spirometer)



(1.0b)

• Typical System Diagram



1. Strain

A. Why measure strain?

Strain is the deformation (e.g. bending) of a material that is under a mechanical load.

Example applications include:

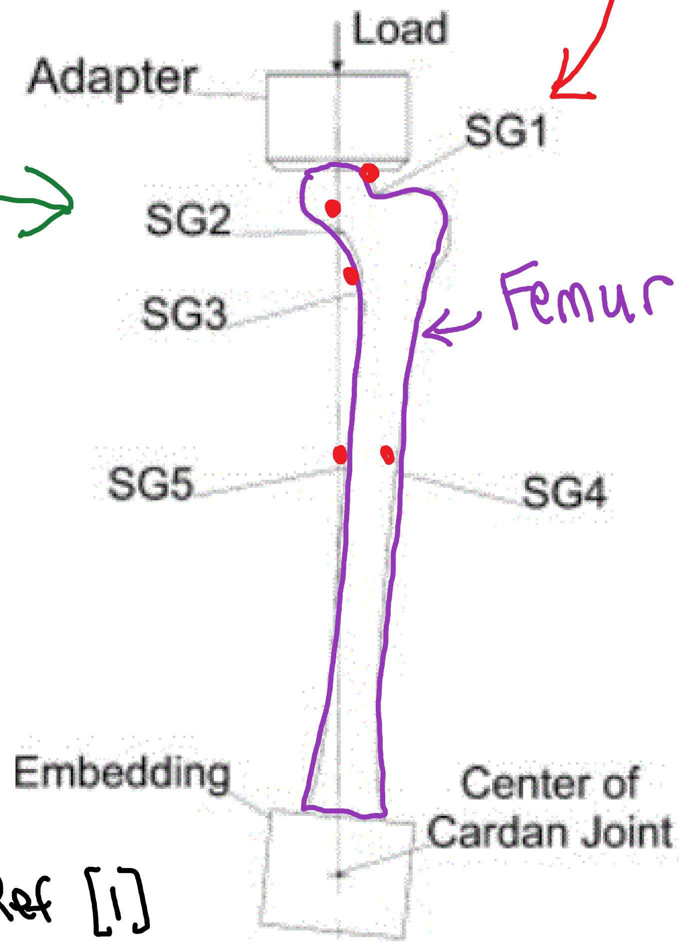
- Bone mechanics
- Soft tissue mechanics
- Prosthesis materials
- Tissue engineering
- Materials testing

Experimental measurement

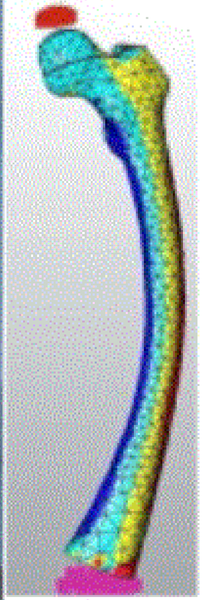
"SG" = Strain Gauge

FEM model of strain

Mechanical Testing



Ref [1]



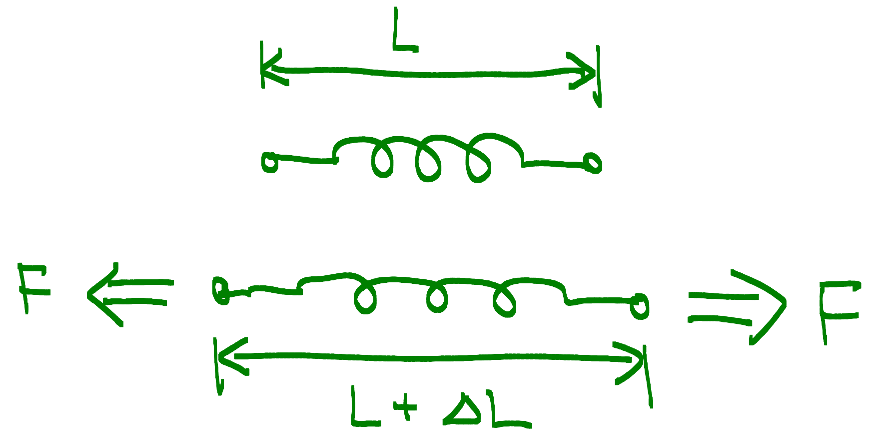
Stretched sample

Ref [2]

B. Stress-Strain Tutorial

- **Hooke's Law**:

For a spring, $F = k\Delta L$ ← spring constant

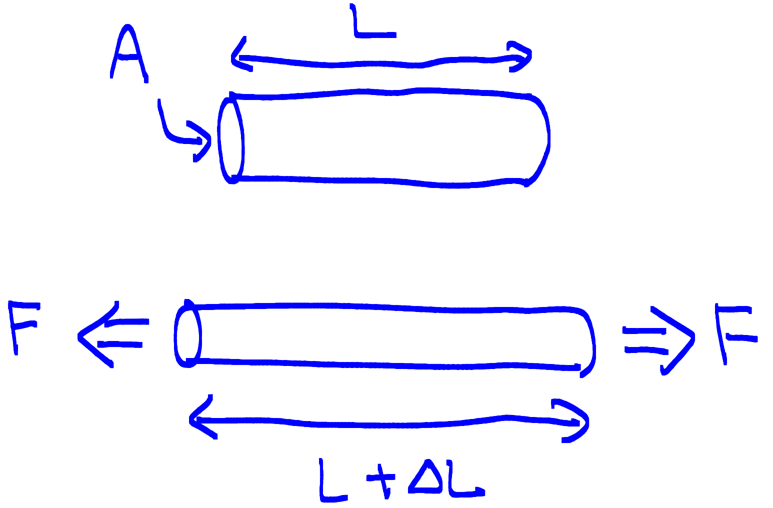


For a bulk material, $\sigma = E\,\varepsilon$

Stress → σ; Elastic Modulus → E; strain → ε

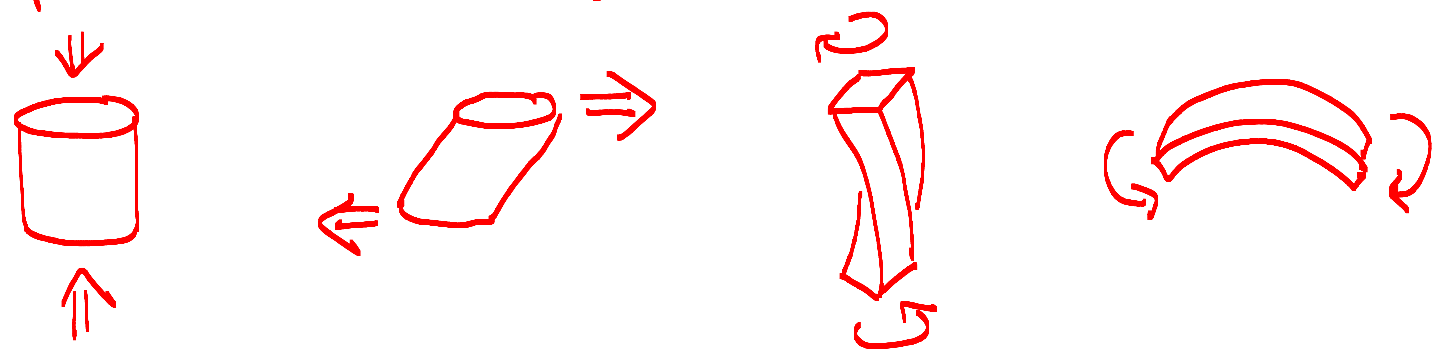
Example: Tensile Strain $\frac{\Delta L}{L}$

$$\boxed{\frac{F}{A} = E\frac{\Delta L}{L}}$$



- Strain can also be

compressive, shear, torsional, flexural, ...



Q: How to measure strain?

→ strain gauge!

2. Strain Gauge

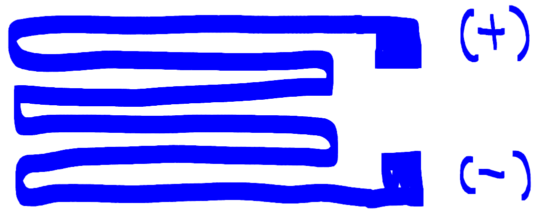
(1.3)

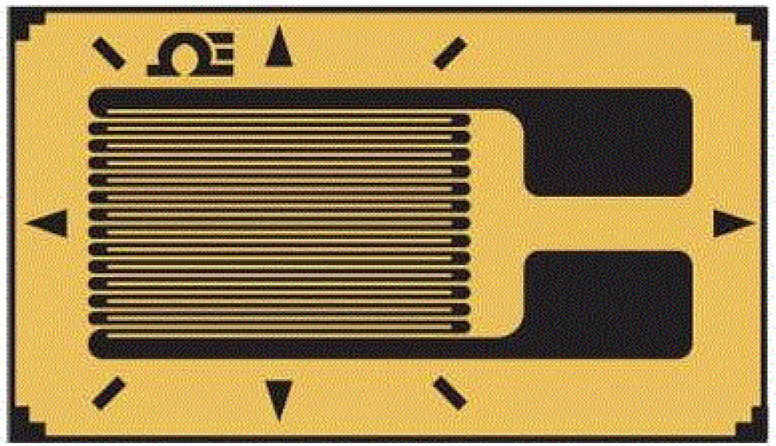
www.omega.com

Ref [3]

When a thin wire is stretched, its electrical resistance increases.

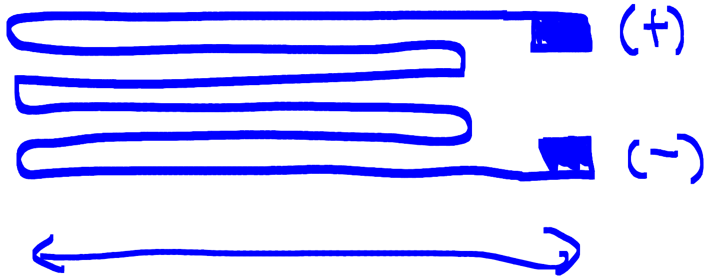
(no strain)





Few mm

$\Rightarrow$ strain dependent resistor!



(Tensile strain)

$$\frac{\Delta R}{R} = G\,\varepsilon$$

Gauge Factor

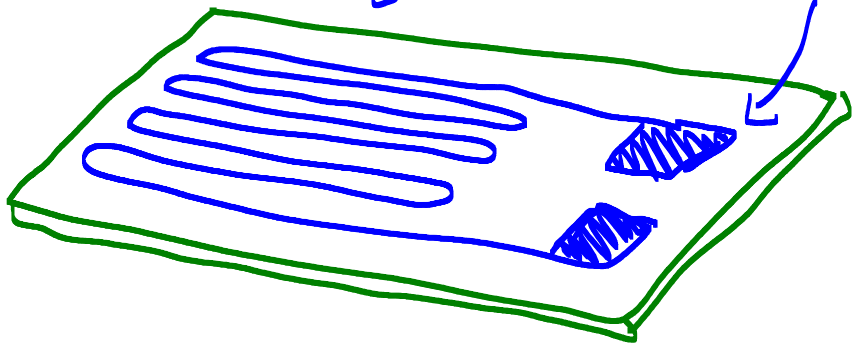
↳ $\varepsilon > 0$ (Tension)

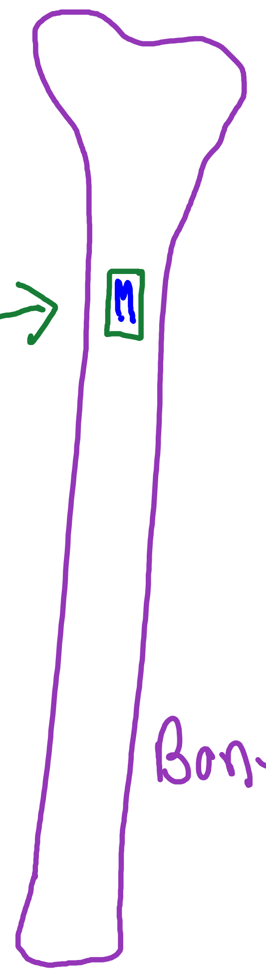
↳ $\varepsilon < 0$ (Compression)

~5 µm thick metal foil pattern

Solder Pad

~25µm thick plastic substrate



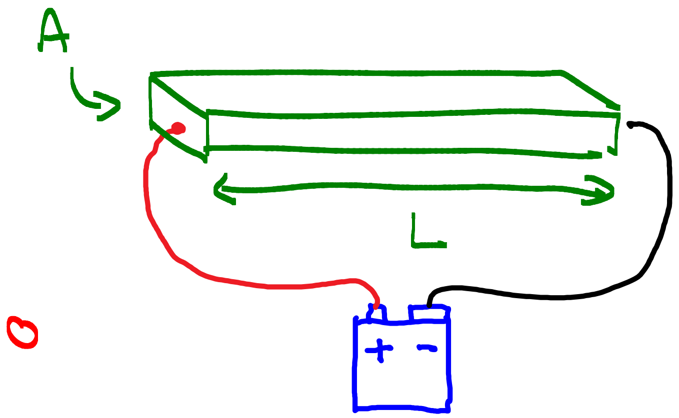


Bone

NOTE: Typical $\varepsilon \sim 10^{-6}$ (microstrain) in solid materials

$\Rightarrow \frac{\Delta R}{R}$ is tiny!

- Derivation of $\frac{\Delta R}{R} = G\varepsilon$: $R = \rho \frac{L}{A}$ ← resistivity



$$\Delta R = \frac{\partial R}{\partial L}\Delta L + \frac{\partial R}{\partial A}\Delta A + \frac{\partial R}{\partial \rho}\Delta \rho$$

$$= \frac{\rho}{A}\Delta L - \rho\frac{L}{A^2}\Delta A + \frac{L}{A}\Delta\rho$$

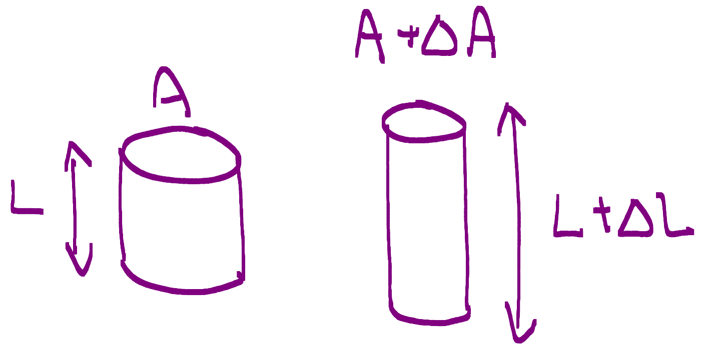
$$= \underbrace{\frac{\rho L}{A}}_{R}\Big[\underbrace{\frac{\Delta L}{L} - \frac{\Delta A}{A}}_{\text{Dimensional Effect}} + \underbrace{\frac{\Delta\rho}{\rho}}_{\text{Piezoresistive Effect}}\Big]$$

$$\frac{\Delta R}{R} = \frac{\Delta L}{L} + 2\overset{\text{Poisson Ratio}}{\mu}\frac{\Delta L}{L} + \frac{\Delta\rho}{\rho}$$

$$= \Big[1 + 2\mu + \underbrace{\frac{\Delta\rho/\rho}{\Delta L/L}}_{\text{Significant in semiconductors}}\Big]\Big[\overset{\varepsilon}{\frac{\Delta L}{L}}\Big] = \underset{\text{Gauge Factor}}{G}\cdot\varepsilon \ ✓$$

Ni/Cu alloy → Common metal is constantan ($G = 2.1$)

$$\frac{\Delta A}{A} = -2\underset{\text{Poisson Ratio}}{\mu}\frac{\Delta L}{L}$$



(1.5)

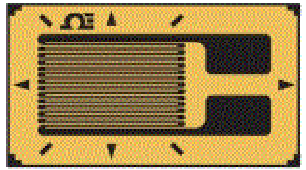
Table 2.1 Properties of Strain-Gage Materials

Material	Composition (%)	Gage Factor	Temperature Coefficient of Resistivity ($°C^{-1}$–10^{-5})
Constantan (advance)	Ni_{45}, Cu_{55}	2.1	±2
Isoelastic	Ni_{36}, Cr_8 $(Mn, Si, Mo)_4$ Fe_{52}	3.52 to 3.6	+17
Karma	Ni_{74}, Cr_{20}, Fe_3 Cu_3	2.1	+2
Manganin	Cu_{84}, Mn_{12}, Ni_4	0.3 to 0.47	±2
Alloy 479	Pt_{92}, W_8	3.6 to 4.4	+24
Nickel	Pure	−12 to −20	670
Nichrome V	Ni_{80}, Cr_{20}	2.1 to 2.63	10
Silicon	(*p* type)	100 to 170	70 to 700
Silicon	(*n* type)	−100 to −140	70 to 700
Germanium	(*p* type)	102	
Germanium	(*n* type)	−150	

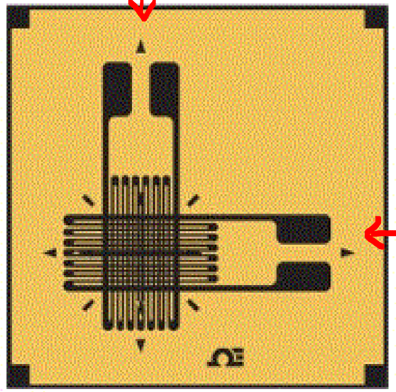
Membrane-type pressure sensors

Strong piezoresistive effect

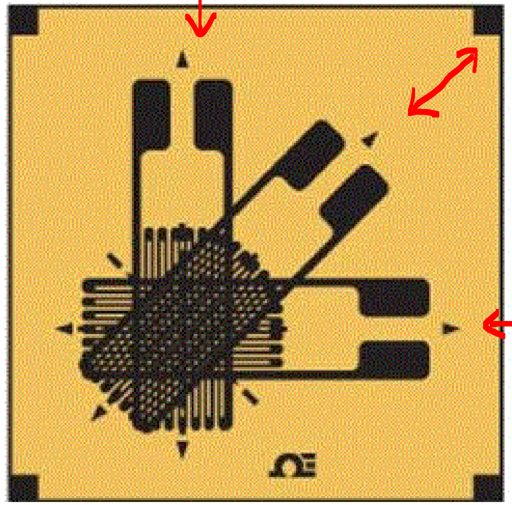
Single Axis (uni-axial)



Biaxial



Rosette



Ref [3] www.omega.com

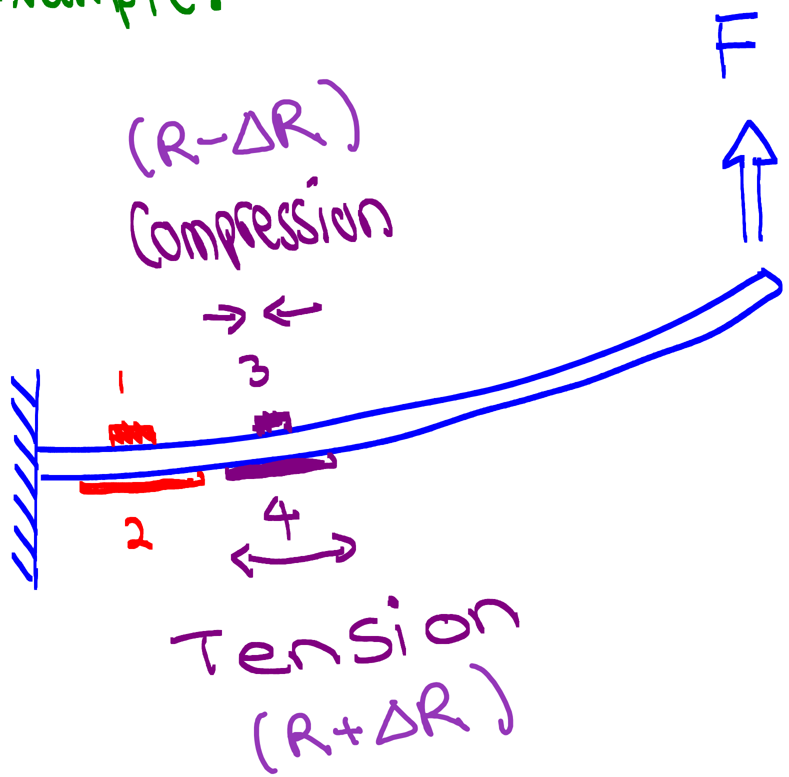
3. Signal Conditioning

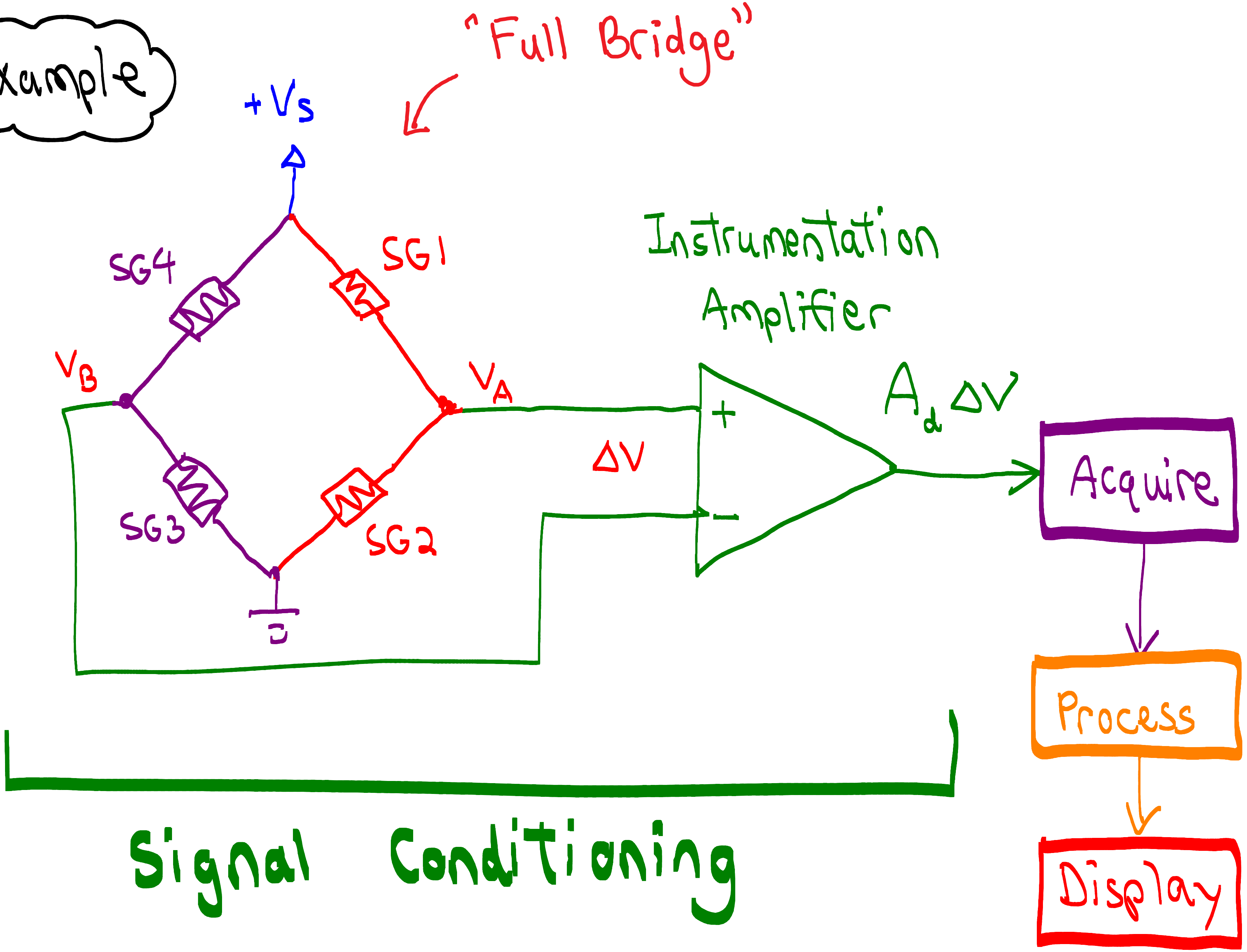
Example

A. How to make a recording?

- To be useful, a strain gauge must be part of a circuit!

Example:

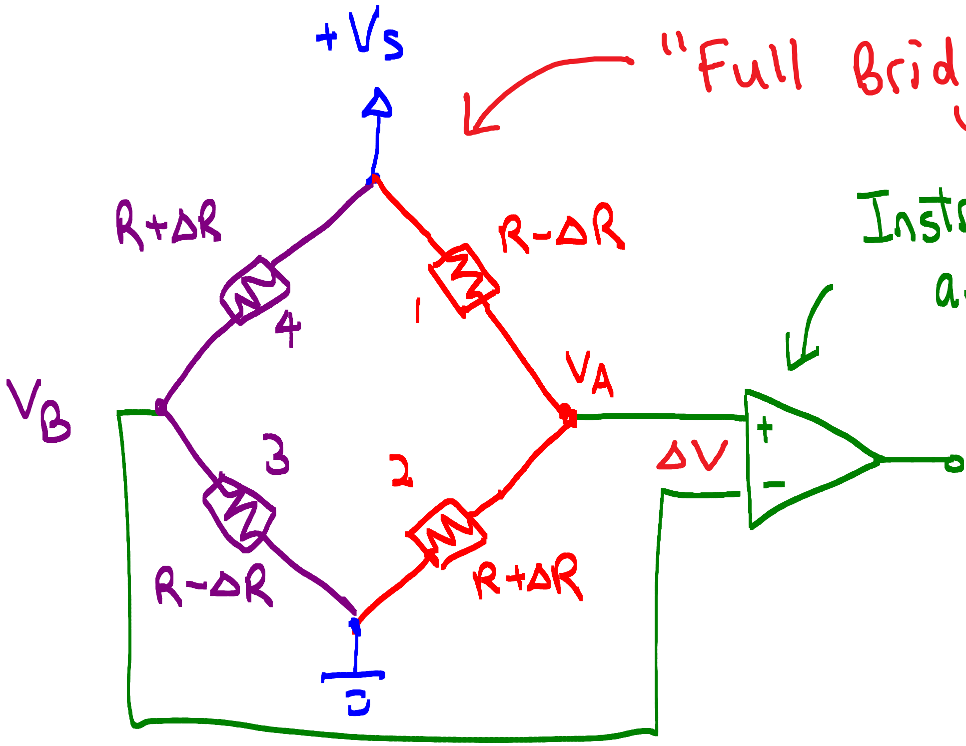




B. Resistor Bridge

- Classic method to detect tiny ΔR (e.g. 0.01Ω)
  - → Bridge has 4 resistor "legs"
  - ⇒ Sensor affects $V_A - V_B$

"Full Bridge" (4 SGs)



Instrumentation amplifier

$$A_d(V_A - V_B)$$
$$= A_d \Delta V$$
$$= A_d V_s \frac{\Delta R}{R} = A_d V_s G \varepsilon$$

- $V_A = V_s \dfrac{R+\Delta R}{(R+\Delta R)+(R-\Delta R)} = V_s \dfrac{R+\Delta R}{2R}$

$$= \boxed{V_s \cdot \tfrac{1}{2}} + \boxed{V_s \frac{\Delta R}{2R}}$$

Const

Tiny change

- $V_B = \boxed{V_s \cdot \tfrac{1}{2}} - \boxed{V_s \frac{\Delta R}{2R}}$

$$\Rightarrow \underbrace{V_A - V_B}_{\Delta V} = 0 + V_s \frac{\Delta R}{R}$$

send this tiny voltage through an amplifier

NOTE: This assumes opposite strain between SG1 and SG2
(SG3) (SG4)

C. Instrumentation Amplifier ← Discuss details later... Ref [4] (1.8)

- Used in many biomedical measurement systems
- Amplifies the difference between two inputs

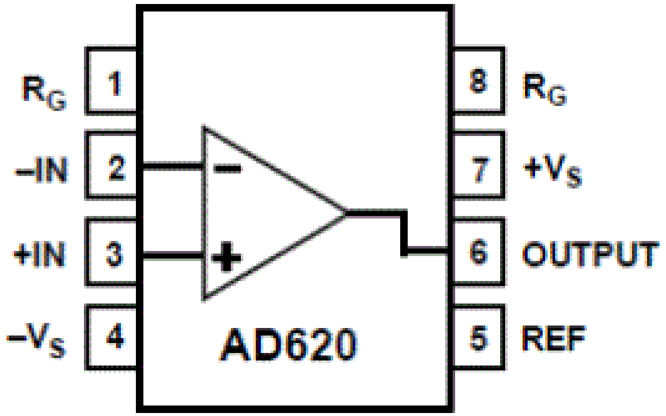
$$V_{OUT} = A_d\,(V_{IN+} - V_{IN-})$$

↑ Differential Gain

ANALOG DEVICES

Low Cost Low Power Instrumentation Amplifier

AD620

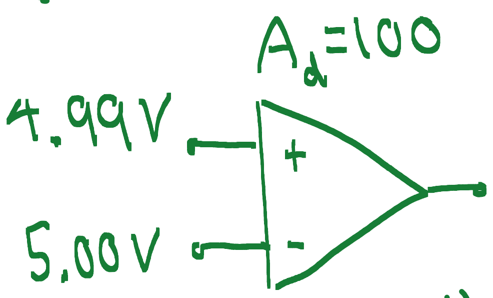


APPLICATIONS

Weigh scales
ECG and medical instrumentation
Transducer interface
Data acquisition systems
Industrial process controls
Battery-powered and portable equipment

we are using this kind, but there are lots of others...

Ex:



$$V_{OUT} = 100 \times (\underbrace{4.99 - 5.00}_{-.01}) = \underline{\underline{-1\,V}}$$

"In Amp"

↑ NOT the same as an op amp

Can be configured to make a variety of circuits (integrator, summing, etc.)

D. Processing

Q: once we measure $A_d \Delta V$, how do we get strain?

Example: For full bridge:

$$V_{measure} = A_d \boxed{\Delta V} = A_d V_s \boxed{\frac{\Delta R}{R}} = A_d V_s G \varepsilon$$

Measure this

we want this!

$\Rightarrow$ use algebra to solve for ε!

$$\Rightarrow \varepsilon = \underbrace{\frac{1}{A_d G V_s}}_{\text{Conversion Factor } (1/\text{Volt})} \cdot V_{meas}$$

data from circuit

Physically meaningful quantity

Example: $V_{measure} = .5V$

$A_d = 500$

$G = 2.1$

$V_s = 5V$

For full bridge:

$$\rightarrow \varepsilon = \frac{1}{500 \cdot 2.1 \cdot 5V} \cdot 0.5V$$

$$= \left(1.9 \times 10^{-4} \frac{1}{V}\right) \cdot 0.5V$$

$$= 9.5 \times 10^{-5} = 95 \times 10^{-6}$$

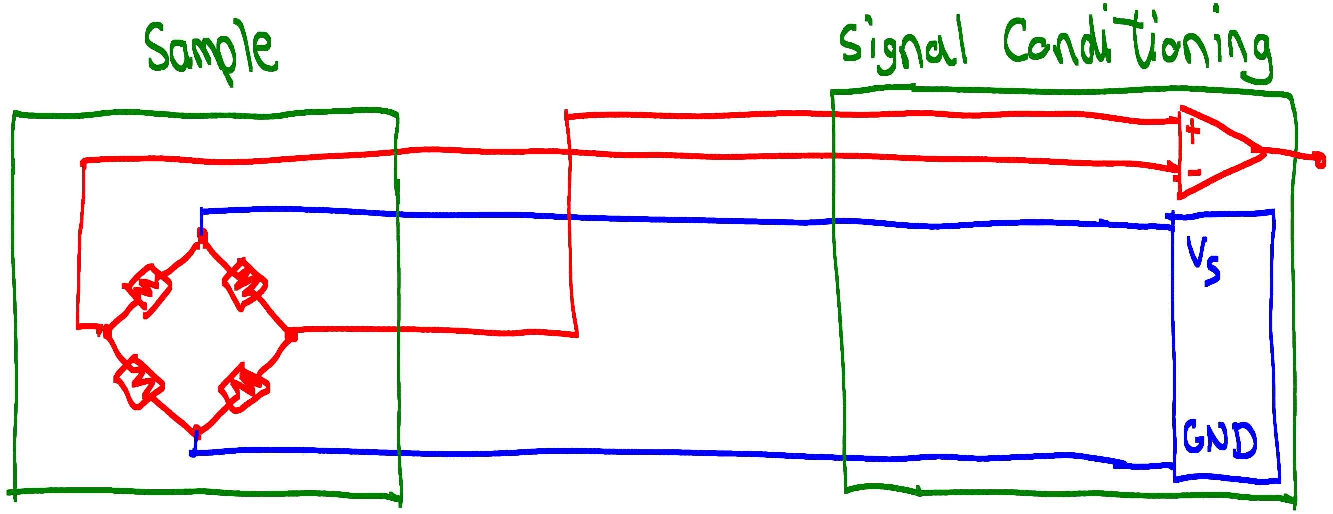
$= \underline{95 \text{ microstrain}}$

- Bridge Types

FULL BRIDGE

4 active bridge legs on the sample

$$\Delta V = V_s \frac{\Delta R}{R}$$

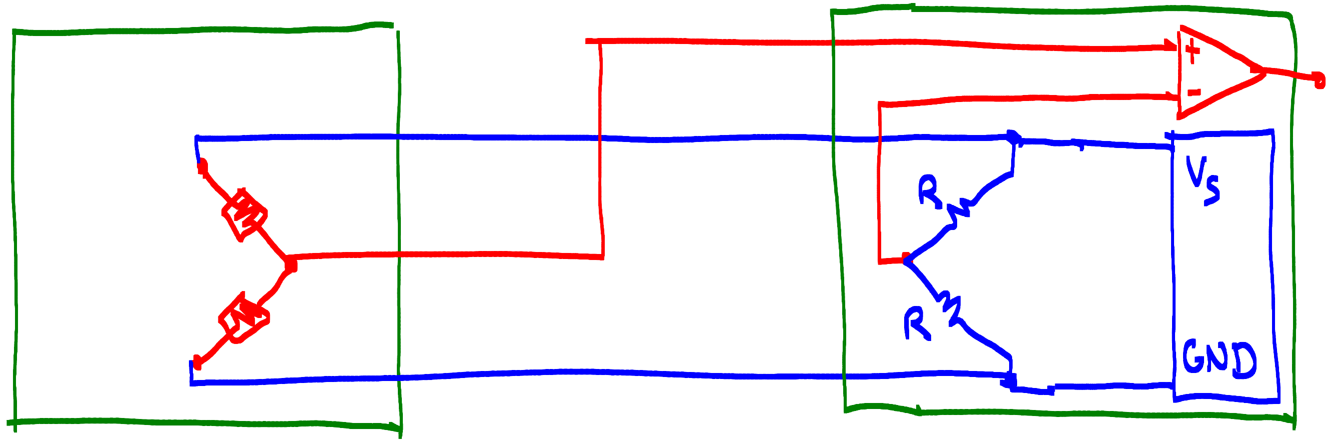


$$OUT = A_d V_s G \varepsilon$$

HALF BRIDGE

2 active bridge legs on the sample

$$\Delta V = \frac{1}{2} V_s \frac{\Delta R}{R}$$

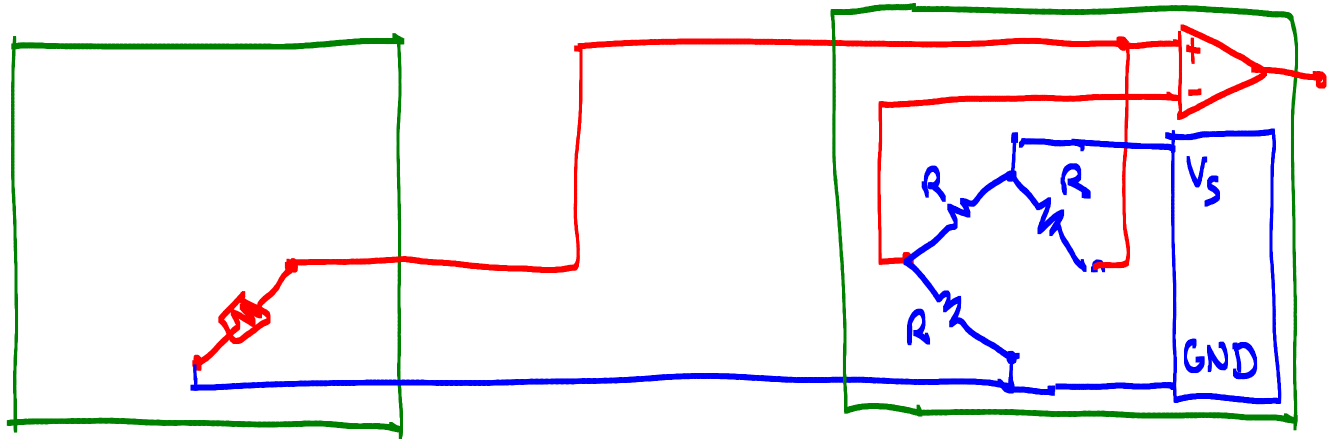


$$OUT = A_d V_s \frac{1}{2} G \varepsilon$$

QUARTER BRIDGE

1 active bridge leg on the sample

$$\Delta V = \frac{1}{4} V_s \frac{\Delta R}{R}$$



$$OUT = A_d V_s \frac{1}{4} G \varepsilon$$

- References

[1] Z. Yoshibash, "p-FEMs in biomechanics: Bones and Arteries", Computer Methods in Applied Mechanics and Engineering, v249-252, pp169-184 (2012)

[2] Instron (www.instron.us)

[3] Omega Engineering, Inc (omega.com)

[4] Analog Devices, Inc (analog.com)