Homework #5

Chapter 5 – X-rays

Questions

Q5.1   There are many ways in which x-rays can interact with matter. The two main
ones that we've discussed are the photoelectric absorption and Compton
scattering. High Z materials are good for shielding

Q5.3   Air was used as a contrast medium compared to brain matter due to the
attenuation coefficients of air and say fatty material. Air attenuates very little
while the brain attenuates more. Using air in the veins/arteries in the brain will
highlight the regions of maximum/minimum attenuation.

Q5.10 One measurement would be insufficient to measure bone density since all of the
absorption coefficients would be unable to be determined. Using DEXA we take
measurements that both involve and do not involve bone. This way we can
compensate for the soft tissue.

Problems

P5.2
a. What fraction of 140 keV x-rays incident upon a 0.5 mm thick lead apron will be
transmitted?
The mass absorption coefficient of lead for 140 keV x-rays is: $\mu_m = 2.0 \text{ cm}^2 \text{ g}^{-1}$
The density of lead is: $\rho = 11.3 \text{ g/cm}^3$.
We can compute the attenuation coefficient for 140 keV x-rays from:
   $\mu = \mu_m \times \rho = 2.0 \text{ cm}^2 \text{ g}^{-1} \times 11.3 \text{ g/cm}^3 = 23 \text{ cm}^{-1}$
We then can use the exponential fall-off of the x-ray intensity to find out how many
x-rays will be transmitted through the apron. Here, x = 0.5 mm = 5×10^{-2} cm.

$$I = I_o \, e^{-\mu x}$$

$$= I_o \, e^{-\left(23 cm^{-1}\right) \times 5 \times 10^{-2} cm}$$

$$= I_o \, e^{-1.15}$$

$$= 0.3 \, I_o$$

$$= 30\% \, I_o$$

So, 30% of the original x-ray intensity is transmitted through the lead apron. See
comments following part b.
b. How thick would the lead shielding in a wall of an x-ray experimental laboratory
have to be to reduce the intensity of 8.0 keV x-rays to 1% of its original value?
The mass absorption coefficient of lead for 8.0 keV x-rays is: $\mu_m = 232 \text{ cm}^2 \text{ g}^{-1}$

Since we know the density of lead from part a), we can compute the attenuation coefficient of lead for 8.0 keV x-rays:

$$\mu = \mu_m \times \rho = 232 \text{ cm}^2 \text{ g}^{-1} \times 11.3 \text{ g/cm}^3 = 2620 \text{ cm}^{-1}$$

We can now compute the relationship between the desired x-ray transmission and the thickness of lead, x, required:

$$I = 1\% I_o = 0.01 I_o$$

$$= I_o \, e^{-\mu x}$$

$$= I_o \, e^{-\left(2620\,cm^{-1}\right)x}$$

or:

$$0.01 = e^{-\left(2620\,cm^{-1}\right)x}$$

We can use the natural logarithm function to extract out the value of x:

$$0.01 = e^{-\left(2620\,cm^{-1}\right)x}$$

$$\ln(0.01) = \ln\left(e^{-\left(2620\,cm^{-1}\right)x}\right)$$

$$-4.6 = -\left(2620\,cm^{-1}\right)x$$

$$x = \frac{4.6}{2620\,cm^{-1}} = 1.8 \times 10^{-3}\,cm$$

(Note: you can always check this answer by using this value of x in the equation for the transmitted intensity, to make sure it really does give you 1% transmission.)
This is a very, very thin piece of lead indeed. Now, we return to the results of part a). Note that shielding low energy x-rays is relatively easy. It requires only a very thin lead foil. The apron from part a) would give excellent shielding of these x-rays. To shield higher energy x-rays--energies corresponding to the high end of those used in diagnostic imaging--requires much thicker lead shielding. A lead apron with the thickness given in part a) would be adequate to shield lower energy x-rays commonly used in imaging, for example, but a thicker layer of lead would be necessary to shield higher energy x-rays

P5.3

a. For the case of the 1cm thick rib, we have x = 1 cm and for 20 keV $m_{bone} = 4.8 \text{ cm}^{-1}$

$$\text{Percent transmission} = 100\% \times e^{-\mu x}$$

$$= 100\% \times e^{-4.8cm^{-1} \times 1\,cm}$$

$$= 0.82\%$$

Note that if we always use the lower value for For 60 keV, $m_{bone} = 0.55 \text{ cm}^{-1}$

$$\text{Percent transmission} = 100\% \times e^{-0.55cm^{-1} \times 1\,cm}$$

$$= 58\%$$

From the text, we learned that 2.1×10^{-7} of the original beam is transmitted by 20 cm of soft tissue alone for 20 keV x-rays; for 60 keV x-rays, 1.5×10^{-2} of the original x-rays are transmitted. We multiply this factor by the two results above to get a total

transmission at 20 keV of $0.82\% \times 2.1 \times 10^{-7} = 1.7 \times 10^{-7}\%$ and at 60 keV of 0.87 %.

b. For the 4 cm region of breast tissue, we have transmissions of:

20 keV: $\dfrac{I_{trans}}{I_o} = e^{-\mu x} = e^{-0.76 cm^{-1} \times 4 cm} = 0.048$

60 keV: $\dfrac{I_{trans}}{I_o} = e^{-\mu x} = e^{-0.20 cm^{-1} \times 4 cm} = 0.43$

As pointed out in the text, more higher energy x-ray photons are transmitted through breast tissue to develop the image receptor. However, as the following problem shows, raising the energy degrades the contrast.

P5.4

a. For the case of the 1 cm thick rib, we have x = 1cm and for 20 keV: $m_2 = 0.76$ cm^{-1} and $m_1 = 4.8$ cm^{-1}

$$\text{Percent difference} = 100\% \times \frac{1 - e^{-(\mu_1 - \mu_2)x}}{1 + R}$$

$$= 100\% \times \frac{1 - e^{-(4.8 - 0.76) cm^{-1} \times 1 cm}}{1 + 0}$$

$$= 92\%$$

Note that we have chosen our values of absorption coefficient such that m_2 is always the lower value. This ensures that the contrast varies between 100% (for very different values of transmission) to 0% (for identical transmission). This choice of values means that the rib is less transmitting than the soft tissue. For 60 keV: $m_2 = 0.20$ cm^{-1} and $m_1 = 0.55$ cm^{-1}

$$\text{Percent difference} = 100 \times \frac{1 - e^{-(0.55 - 0.20) cm^{-1} \times 1 cm}}{1 + 0}$$

$$= 30\%$$

Again, the ordering of absorption coefficients reflects the fact that the bone is less transmitting/more absorbing than the soft tissue.

b. Now the thickness, x, is still 1 cm and for air, $m_2=0$ always. For 20 keV: $m_1 = 0.76$ cm^{-1}

$$\text{percent difference} = 100 \times \frac{1 - e^{-(0.76 - 0) cm^{-1} \times 1 cm}}{1 + 0}$$

$$= +53\%$$

For 60 keV: $m_1 = 0.20$ cm^{-1}

$$\text{Percent difference} = 100 \times \frac{1 - e^{-(0.20 - 0) cm^{-1} \times 1 cm}}{1 + 0}$$

$$= +18\%$$

Both contrast values indicate that more transmission occurs for the air sac. For 20 keV a greater increase is seen than at 60 keV, again indicating the increased contrast at lower energies.

P5.5

a. For the case of the microcalcification, we have $x = 0.1$ mm $= 10^{-2}$ cm and for 20 keV: $m_1 = 0.6$ cm^{-1} and $m_2 = 4.8$ cm^{-1}

$$C = \frac{I_1 - I_2}{I_1}$$
$$= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right)$$
$$= 100\% \times \left(1 - e^{-(4.8-0.5)\text{cm}^{-1} \times 10^{-2}\ \text{cm}}\right)$$
$$= 100\% \times \left(1 - e^{-0.043}\right)$$
$$= 4\%$$

I'll comment on this result after doing the calculation for part b).

b. For 60 keV: $m_1 = 0.17$ cm^{-1} and $m_2 = 0.55$ cm^{-1} for mineralized bone, used to model the microcalcification's absorption coefficient. (Here I've chosen the greatest difference in attenuation coefficients to give the best possible contrast. You could have chosen the other value of 0.47 to be conservative.) The contrast, C, or percentage difference in x-ray intensity transmitted, is given by:

$$C = 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right)$$
$$= 100\% \times \left(1 - e^{-(0.55-0.17)\text{cm}^{-1} \times 10^{-2}\ \text{cm}}\right)$$
$$= 100\% \times \left(1 - e^{-0.0038}\right)$$
$$= 3.8 \times 10^{-3} = 0.38\%$$

The positive sign in both cases means that the microcalcification is less transmitting/more absorbing than fat, so that I_1 (the transmission through fat alone) is greater than I_2 (the transmission through fat plus the microcalcification); for 20 keV, a much greater difference in transmission occurs for the two tissues than is the case for 60 keV x-rays. In fact, only an image made with 20 keV x-rays would be able to distinguish the microcalcification given an x-ray film/phosphor combination only sensitive to contrasts greater than about 1%.

c. Now the thickness, $x = 0.1$ cm. For 20 keV: $m_1 = 0.5$ cm^{-1} and $m_2 = 0.76$ cm^{-1}

$$C = \frac{I_1 - I_2}{I_1}$$
$$= 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right)$$
$$= 100\% \times \left(1 - e^{-(0.76-0.5)\text{cm}^{-1} \times 10^{-1}\ \text{cm}}\right)$$
$$= 100\% \times \left(1 - e^{-0.026}\right)$$
$$= 2.6\%$$

See comments below after the calculation for part d)

d. For 60 keV: For 20 keV: $m_1 = 0.17$ cm^{-1} and $m_2 = 0.20$ cm^{-1}

$$C = 100\% \times \left(1 - e^{-(\mu_2 - \mu_1)x}\right)$$
$$= 100\% \times \left(1 - e^{-(0.20 - 0.17)\text{cm}^{-1} \times 10^{-1} \text{ cm}}\right)$$
$$= 100\% \times \left(1 - e^{-0.0030}\right)$$
$$= 3 \times 10^{-3}$$
$$= 0.3\%$$

For 20 keV the contrast is again significantly higher than for 60 keV, and only the 20 keV case would be detectable on the film/phosphor combination.

Thus, we see that for both of these cases, only the 20 keV case would correspond to a detectable image, even neglecting the effects of scattering, noise, etc. Thus, although the 20 keV x-rays provide a higher x-ray dose than would 60 keV x-rays, they are essential for imaging the possible signs of a tumor. In both cases, we see that these numbers come out close to the limits of detectability anyway, showing that microcalcifications smaller than 0.1 mm are considered to be undetectable with most current mammography setups, and that small solid tumors are also difficult to distinguish. Earlier mammography systems used in the early 1970's were actually unable to perform at this level, and consequently did not provide adequate mammograms for detecting early breast cancer. It is hoped that ongoing improvements in x-ray imaging will lead to even better detection rates, *improving* the rates of breast cancer detection and cures relative to those observed in the population studies to date. (In fact, these studies could not really assess improvements available since the mid-1980's, since not enough time has elapsed since then to evaluate their effectiveness.)

P5.8

a. *What is the operating voltage of the tube? What is the kVp?* These two quantities are actually based on the same quantity--the kVp is determined by the tube's operating voltage, because it represents the maximum amount of energy an electron has after crossing the voltage difference between the tube's cathode and anode. From the plot, we see that the maximum energy of the x-rays produced is approximately 87 keV. This is also the maximum energy of the electrons in the tube, hence the kVp and operating voltages are 87 kV.

b. *What is the anode material?* It is clear that the peak locations in the plot provided for this problem coincide exactly with the characteristic emission lines for tungsten. This agreement of the characteristic x-ray emission lines identifies the anode material as tungsten.

c. *Explain which features of the curve correspond to brehmsstrahlung and which to characteristic x-rays.* The broad, continuous spectrum of emitted x-rays correspond to brehmsstrahlung--this process is not selective in the energies of x-rays produced; the sharp emission peaks correspond to characteristic x-ray emission--these peak locations are sensitively determined by the exact atomic composition of the anode material. d) *How would the curve be changed qualitatively if the operating voltage of the tube were halved?* In that case, the kVp would move to half of its present value. *If the operating current were doubled?* The number of x-rays produced would

double also if the number of electrons hitting the tube were doubled by doubling the electrical current.