Physics 111 Homework Solutions Week #9 - Thursday

Monday, March 1, 2010
Chapter 24
Questions
Questions

24.1 Based on special relativity we know that as a particle with mass travels near the speed of light its mass increases. In order to accelerate this object from rest to a speed near that of light would require an ever increasing force (one that rapidly becomes larger by a factor of γ.) There are no known forces that could accelerate a particle with mass to the speed of light in a finite amount of time and with a finite amount of energy. So for objects with no rest mass, as they travel at the speed of light, there mass does not increase with increasing speed and we avoid these problems of accelerating the massless particles.

24.6 The Compton shift in wavelength for the proton and the electron are given by $\Delta\lambda_e = \frac{h}{m_e c}(1-\cos\phi)$ and $\Delta\lambda_p = \frac{h}{m_p c}(1-\cos\phi)$ respectively. Evaluating the ratio of the shift in wavelength for the proton to the electron, evaluated at the same detection angle ϕ, we find $\frac{\Delta\lambda_p}{\Delta\lambda_e} = \frac{hm_e c}{m_p ch} = \frac{m_e}{m_p} = 5\times10^{-4} \rightarrow \Delta\lambda_p = 5\times10^{-4}\Delta\lambda_e$. Therefore the shift in wavelength for the proton is smaller than the wavelength shift for the electron.

Multiple-Choice

- None

Problems

Relativistic energy and momentum for an object of mass m.

For an object with a $m = 1kg$ rest mass, it has a rest energy of $E = mc^2 = 9x10^{16}\ J$. The Lorentz factor is given by: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, with the relativistic momentum $p = \gamma mv$, and the relativistic energy $E^2 = p^2c^2 + m^2c^4$.

a. For a velocity of 0.8c, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.64}} = 1.67$. The relativistic momentum and energy are therefore,

$p = \gamma mv = 1.67\times1kg\times0.8\times3\times10^8\ \tfrac{m}{s} = 4.0\times10^8\ \tfrac{kgm}{s}$ and

$E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{\left(4.0\times10^8\ \tfrac{kgm}{s}\times3\times10^8\ \tfrac{m}{s}\right)^2 + \left(9\times10^{16}\ J\right)^2} = 1.5\times10^{17}\ J$

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b. Following the procedure in part a, for a velocity of 0.9c, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.81}} = 2.29$, the relativistic momentum is *6.19x10*^{8} *kgm/s*, and the relativistic energy is *2.07x10*^{17} *J*.

c. For a velocity of 0.95c, γ = 3.20, the relativistic momentum is *9.13x10*^{8} *kgm/s*, and the relativistic energy is *2.88x10*^{17} *J*.

d. For a velocity of 0.99c, γ = 7.09, the relativistic momentum is *2.11x10*^{9} *kgm/s*, and the relativistic energy is *6.387x10*^{17} *J*.

e. For a velocity of 0.999c, γ = 22.4, the relativistic momentum is *6.70x10*^{9} *kgm/s,* and the relativistic energy is *2.0x10*^{18} *J*.

24.2 For an electron with rest mass *9.11x10*$^{-31}$ *kg*, it has a rest energy of $E = mc^2$ = *8.199x10*$^{-14}$ *J = 0.511 MeV*. The Lorentz factor is given by: $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and the relativistic momentum and energy are given respectively as $p = \gamma mv$ and $E = \sqrt{p^2c^2 + m^2c^4}$.

a. For the electron with a velocity of *0.8c, γ = 1.67*. The relativistic momentum and energy are therefore,

$p = \gamma mv = 1.67 \times 9.11 \times 10^{-31} kg \times 0.8 \times 3 \times 10^{8} \frac{m}{s} = 3.65 \times 10^{-22} \frac{kgm}{s}$ and

$E = \sqrt{p^2c^2 + m^2c^4} = \sqrt{\left(3.65 \times 10^{-22} \frac{kgm}{s} \times 3 \times 10^{8} \frac{m}{s}\right)^2 + \left(8.199 \times 10 - 14J\right)^2} = 1.37 \times 10^{-13} J$

Using the fact that *1.6x10*$^{-19}$ *J = 1 eV*, the relativistic energy is given as $E = 1.37 \times 10^{-13} J \times \frac{1eV}{1.6 \times 10^{-19} J} = 0.855 \times 10^{6} eV = 0.855 MeV$. Further it can be shown that the total relativistic energy is given as $E = \gamma mc^2 = \gamma E_{rest}$. Thus the relativistic energy can be computed in a more efficient method. $E = \gamma E_{rest} = 1.67 \times 0.511 MeV = 0.855 MeV$.

b. Following the method outlined in part a, for a velocity of *0.9c, γ = 2.29*. The relativistic momentum is therefore,

$p = \gamma mv = 2.29 \times 9.11 \times 10^{-31} kg \times 0.9 \times 3 \times 10^{8} \frac{m}{s} = 5.63 \times 10^{-22} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 2.29 \times 0.511 MeV = 1.17 MeV$.

c. For a velocity of *0.95c, γ = 3.2*. The relativistic momentum is therefore,

$p = \gamma mv = 3.2 \times 9.11 \times 10^{-31} kg \times 0.95 \times 3 \times 10^{8} \frac{m}{s} = 8.31 \times 10^{-22} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 3.2 \times 0.511 MeV = 1.63 MeV$.

d. For a velocity of *0.99c, γ = 7.09*. The relativistic momentum is therefore,

$p = \gamma mv = 7.09 \times 9.11 \times 10^{-31} kg \times 0.99 \times 3 \times 10^{8} \frac{m}{s} = 1.92 \times 10^{-21} \frac{kgm}{s}$ and

$E = \gamma E_{rest} = 7.09 \times 0.511 MeV = 3.62 MeV$.

e. For a velocity of *0.999c, γ = 22.4*. The relativistic momentum is therefore,

$p = \gamma m v = 22.4 \times 9.11 \times 10^{-31} kg \times 0.999 \times 3 \times 10^{8} \frac{m}{s} = 6.12 \times 10^{-21} \frac{kgm}{s}$ and
$E = \gamma E_{rest} = 22.4 \times 0.511 MeV = 11.45 MeV$.

From equation (2) we have the kinetic energy given by $KE_{rel} = \gamma mc^2 - mc^2 = (\gamma - 1)mc^2$. Using the fact that $v << c$, we can use the binomial expansion on the Lorentz factor for small arguments. This gives $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = (1 - \frac{v^2}{c^2})^{-1/2} \approx 1 - (-\frac{1}{2}\frac{v^2}{c^2}) = 1 + \frac{1}{2}\frac{v^2}{c^2}$.

Therefore the equation for the kinetic energy becomes
$KE_{rel} = \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)mc^2 - mc^2 = mc^2 + \frac{1}{2}\frac{v^2}{c^2}mc^2 - mc^2 = \frac{1}{2}mv^2$ which is the classical expression for kinetic energy.

24.4 – Will be posted once all of the homework has been turned in.

24.7 For a $1.2 \times 10^{6} eV \times \frac{1.6 \times 10^{-19} J}{1 eV} = 1.92 \times 10^{-13} J$ photon,

a. its momentum is given by $p = \frac{E}{c} = \frac{1.92 \times 10^{-13} J}{3 \times 10^{8} \frac{m}{s}} = 6.4 \times 10^{-22} \frac{kgm}{s}$.

b. its wavelength is given by the de Broglie relation
$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} Js}{6.4 \times 10^{-22} \frac{kgm}{s}} = 1.04 \times 10^{-12} m$.

c. its frequency is given by $f = \frac{c}{\lambda} = \frac{3 \times 10^{8} \frac{m}{s}}{1.04 \times 10^{-12} m} = 2.9 \times 10^{20} s^{-1}$.

24.13 The relativistic energy is given as $E_{rel} = \sqrt{p^2c^2 + m^2c^4}$. The rest energy, mc^2 is $8.199x10^{-19}$ *J = 0.511 MeV*. From the de Broglie wavelength we can calculate the momentum of the electron, $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34} Js}{0.0012 \times 10^{-9} m} = 5.525 \times 10^{-22} kg \frac{m}{s}$. Thus the relativistic energy is
$E_{rel} = \sqrt{\left(5.525 \times 10^{-22} kg \frac{m}{s}\right)^2 \left(3 \times 10^{8} \frac{m}{s}\right)^2 + \left(8.199\text{x}10^{-19}\,\text{J}\right)^2} = 1.657 \times 10^{-13} J = 1.04 MeV.$

Wednesday, March 3, 2010
Chapter 24
Questions
- None

Multiple-Choice
- None

Problems

24.11 A Compton Effect experiment

a. The wavelength and momentum of the incident gamma ray are given as

$$E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}Js)(3\times10^{8}\,\tfrac{m}{s})}{1.6\times10^{6}eV\times\frac{1.6\times10^{-19}J}{1eV}} = 7.77\times10^{-13}m \text{ and}$$

$$p = \frac{h}{\lambda} = \frac{E}{c} = \frac{6.63\times10^{-34}Js}{7.77\times10^{-13}m} = 8.53\times10^{-22}\,\tfrac{kgm}{s} \text{ respectively.}$$

b. Using the Compton formula for the wavelength of the scattered photon and the fact that the energy is inversely proportional to the wavelength we can write

$$\lambda' = \lambda + \frac{h}{m_e c}(1-\cos\phi) \rightarrow \frac{\lambda'}{hc} = \frac{\lambda}{hc} + \frac{1}{m_e c^2}(1-\cos\phi) \rightarrow \frac{1}{E'} = \frac{1}{E} + \frac{(1-\cos\phi)}{m_e c^2}$$

.

c. The energy of the scattered gamma ray photon is

$$\frac{1}{E'} = \frac{1}{1.6MeV} + \frac{(1-\cos 50)}{0.511MeV} = 1.324MeV^{-1} \rightarrow E' = 0.755MeV$$ where the rest mass of the electron is given as

$$m_e c^2 = (9.11\times10^{-31}kg)(3\times10^{8}\,\tfrac{m}{s})^2 \times \frac{1eV}{1.6\times10^{-19}J} = 0.511MeV.$$

d. The kinetic energy of the recoiling electron is found using conservation of energy where when the incident gamma ray photon interacts with electrons in the sample, the gamma ray photon loses some energy to the electron as it scatters. Thus the kinetic energy of the recoiling electron is

$$E_{incident} = E_{scattered} + KE_{e^-}$$

$$\therefore KE_{e^-} = E_{incident} - E_{scattered} = 1.6MeV - 0.755MeV = 0.845MeV$$

e. The speed of the recoiling electron is given by using the expression for the relativistic kinetic energy. We have

$$KE = 0.845MeV = (\gamma - 1)m_e c^2 = (\gamma - 1)0.511MeV$$

$$\gamma = 1 + \frac{0.845MeV}{0.511MeV} = 1 + 1.654 = 2.654 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \rightarrow v^2 = 0.858c^2 \rightarrow v = 0.926c$$

24.12 X-rays on a foil target

a. The energy is given by

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}Js)(3\times10^{8}\,\tfrac{m}{s})}{0.012\times10^{-9}m} = 1.66\times10^{-14}J = 0.104MeV.$$

b. The scattered wavelength is given by

$$\lambda_f - \lambda_i = \frac{h}{mc}(1-\cos\phi) = \frac{2h}{mc} = \frac{2(6.63\times10^{-34}Js)}{(9.11\times10^{-31}kg)(3\times10^{8}\,\tfrac{m}{s})} = 4.86\times10^{-12}m.$$

Thus, $\lambda_f = 4.86\times10^{-12}m + 1.2\times10^{-11}m = 1.686\times10^{-11}m$. The energy is given by the formula in part a, and is $1.18\text{x}10^{-14}$ J = 0.0741 MeV.

c. The energy given to the foil is

$$\Delta E_{f\,oil} = E_{incident} = E_{backscattered} = 0.104MeV - 0.0741MeV = 0.03MeV = 30keV.$$