

Physics 111

Fall 2007

Radioactive Decay Problems Solutions

1. The ${}^3_1\text{H}$ isotope of hydrogen, which is called *tritium* (because it contains three nucleons), has a half-life of 12.33 yr. It can be used to measure the age of objects up to about 100 yr. It is produced in the upper atmosphere by cosmic rays and brought to Earth by rain. As an application, determine approximately the age of a bottle of wine whose ${}^3_1\text{H}$ radiation is about $\frac{1}{10}$ that present in new wine.

Because the tritium in water is being replenished, we assume that the amount is constant until the wine is made, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n;$$
$$0.10 = \left(\frac{1}{2}\right)^n, \text{ or } n \log 2 = \log(10) \text{ which gives } n = 3.32.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (3.32)(12.33 \text{ yr}) = \boxed{41 \text{ yr.}}$$

2. Strontium-90 is produced as a nuclear fission product of uranium in both reactors and atomic bombs. Look at its location in the periodic table to see what other elements it might be similar to chemically, and tell why you think it might be dangerous to ingest. It has too many neutrons, and it decays with a half-life of about 29 yr. How long will we have to wait for the amount of ${}^{90}_{38}\text{Sr}$ on the Earth's surface to reach 1% of its current level, assuming no new material is scattered about? Write down the decay reaction, including the daughter nucleus. The daughter is radioactive: write down its decay.

We see from the periodic chart that Sr is in the same column as $\boxed{\text{calcium}}$.

If strontium is ingested, the body will treat it chemically as if it were calcium, which means it will be $\boxed{\text{stored by the body in bones.}}$

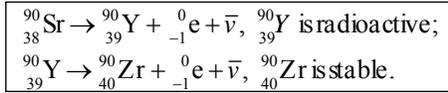
We find the number of half-lives to reach a 1% level from

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n;$$
$$0.01 = \left(\frac{1}{2}\right)^n, \text{ or } n \log 2 = \log(100), \text{ which gives } n = 6.64.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (6.64)(29 \text{ yr}) = \boxed{193 \text{ yr.}}$$

The decay reactions are



3. An old wooden tool is found to contain only 6.0% of ${}^{14}_6\text{C}$ that a sample of fresh wood would. How old is the tool?

Since the carbon is being replenished in living trees, we assume that the amount of ${}^{14}\text{C}$ is constant until the wood is cut, and then it decays. We find the number of half-lives from

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n;$$

$$0.060 = \left(\frac{1}{2}\right)^n, \text{ or } n \log 2 = \log(16.7) \text{ which gives } n = 4.06.$$

Thus the time is

$$t = nT_{\frac{1}{2}} = (4.06)(5730 \text{ yr}) = \boxed{2.3 \times 10^4 \text{ yr.}}$$

4. An amateur archeologist finds a bone that she believes to be from a dinosaur and she sends a chip off to a laboratory for ${}^{14}\text{C}$ dating. The lab finds that the chip contains 5g of carbon and has an activity of 0.5 Bq. How old is the bone and could it be from a dinosaur?

The activity when the bone chip is measured is 0.5 decays/sec. The initial activity when the animal died needs to be determined. In the bone there is found 5g of carbon. Since 1 mole of carbon contains 6.02×10^{23} atoms and 1 mole of carbon has a mass of 12 g, there are 2.51×10^{23} carbon nuclei. Further the ration of ${}^{14}\text{C}/{}^{12}\text{C}$ has remained relatively constant and has a value of 1.3×10^{-12} . Thus the number of ${}^{14}\text{C}$ nuclei is given as $(1.3 \times 10^{-12})(2.51 \times 10^{23} \text{ nuclei}) = 3.26 \times 10^{11}$ ${}^{14}\text{C}$ nuclei when the animal died. The initial activity is a product of the decay constant and the number of ${}^{14}\text{C}$ nuclei present when the animal died. The decay constant is found from the half-life of carbon (5730yrs).

$$\lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{5730 \text{ yrs}} \times \frac{1 \text{ yr}}{3.2 \times 10^7 \text{ sec}} = 3.78 \times 10^{-12} \text{ sec}^{-1}. \text{ The initial activity is } \lambda N =$$

$(3.78 \times 10^{-12} \text{ sec}^{-1})(3.26 \times 10^{11} \text{ } {}^{14}\text{C} \text{ nuclei}) = 1.23 \text{ Bq.}$ To calculate the age of the bone we use

the radioactive decay law

$$A = A_0 e^{-\lambda t} \rightarrow 0.5Bq = 1.23Bqe^{-(3.78 \times 10^{-12} \text{ sec}^{-1})t} \rightarrow \ln\left(\frac{0.5}{1.23}\right) = -(3.78 \times 10^{-12} \text{ sec}^{-1})t$$

$$\rightarrow -0.902 = -(3.78 \times 10^{-12} \text{ sec}^{-1})t \rightarrow t = 2.39 \times 10^{11} \text{ sec} = 7456 \text{ yrs.}$$

Since the bone is only about 7500 years old and knowing that the dinosaurs disappeared over 65 million years ago, it is probably not the bone of a dinosaur.

5. Calculate the decay energy for the β^- decay of ^{24}Na given the following data: $m(^{24}\text{Na}) = 23.98492 \text{ u}$, $m(^{24}\text{Mg}) = 23.97845 \text{ u}$, $m(^{24}\text{Ne}) = 23.98812 \text{ u}$, $m(\beta^-) = 5.49 \times 10^{-4} \text{ u}$. What is the range of the possible energies of the emitted beta particle?

For the β^- -decay reaction of ^{24}Na ,

$$Q = \left(M_{^{24}_{11}\text{Na}} - M_{^{24}_{12}\text{Mg}} - M_{\beta^-} \right) c^2 = \left(23.98492 - 23.97845 - 5.49 \times 10^{-4} \right) \text{ u} c^2 \times \frac{931.5 \text{ MeV}}{\text{u} c^2} = 5.52 \text{ MeV.}$$

6. Show that in alpha decay from a stationary parent nuclide that the conservation of energy and momentum lead to a relation between the decay energy for the nuclear reaction and the kinetic energy gained by the alpha particle, KE, given

$$\text{by } Q = KE \left(1 + \frac{m(^4_2\text{He})}{m(\text{daughter})} \right). \text{ What is the kinetic energy of the alpha particle emitted}$$

in the decay of ^{238}U ?

For alpha decay: $Q = \left(M_{\text{parent}} - M_{\text{daughter}} - M_{\text{He}} \right) c^2$. If the parent is at rest when it decays, then from conservation of momentum, the daughter gets a recoil velocity in the direction opposite direction to the velocity of the alpha particle. Conservation of momentum gives:

$$0 = -m_{\text{daughter}} v_{\text{daughter}} + m_{\alpha} v_{\alpha} \rightarrow v_{\text{daughter}} = \frac{m_{\alpha}}{m_{\text{daughter}}} v_{\alpha}. \text{ Next we apply conservation of}$$

energy and we find:

$$m_{\text{parent}}c^2 = \frac{1}{2}m_{\text{daughter}}v_{\text{daughter}}^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 + m_{\text{daughter}}c^2 + m_{\text{He}}c^2.$$

Next we can replace the velocity of the recoiling daughter atom in terms of the velocity of the alpha particle which can be measured.

$$m_{\text{parent}}c^2 = \frac{1}{2}m_{\text{daughter}}\left(\frac{m_{\alpha}}{m_{\text{daughter}}}v_{\alpha}\right)^2 + \frac{1}{2}m_{\text{He}}v_{\text{He}}^2 + m_{\text{daughter}}c^2 + m_{\text{He}}c^2.$$

Bringing all of the rest energy terms to one side and combining the terms involving the velocity of the alpha particle, we find that

$$(m_{\text{parent}}c^2 - m_{\text{daughter}}c^2 - m_{\text{He}}c^2) = Q = \left[\frac{1}{2}\frac{m_{\alpha}^2}{m_{\text{daughter}}} + \frac{1}{2}m_{\alpha}\right]v_{\alpha}.$$

Factoring out the mass of the alpha particle on the LHS we have :

$$Q = \frac{1}{2}m_{\alpha}v_{\alpha}^2\left[1 + \frac{m_{\alpha}}{m_{\text{daughter}}}\right], \text{ which is the desired result.}$$

Looking up the decay energy, $Q = 4.28 \text{ MeV}$, $m_{\alpha} = 4.0015u$, $m_{\text{daughter}} = m_{\text{thorium}} = 233.99409u$, and we find the velocity of the alpha particle to be $1.42 \times 10^7 \text{ m/s}$.

7. Calculate the binding energies of radium-226 ($m = 225.97709 \text{ u}$), radium-228 ($m = 227.98275 \text{ u}$), and thorium-232 ($m = 231.98864 \text{ u}$).

The nuclear binding energy is given through:

$$NBE = Zm_p c^2 + Nm_n c^2 - m_{\text{atom}} c^2; \text{ where } m_p = 1.00727u \text{ and } m_n = 1.00867u.$$

For radium-226:

$$NBE = [88(1.00727u) + 138(1.00867u) - 225.97709u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1731.8\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1731.8\text{MeV}}{226} = 7.66 \frac{\text{MeV}}{\text{nucleon}}.$$

For radium-228:

$$NBE = [88(1.00727u) + 140(1.00867u) - 227.98275u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1742.7\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1742.7\text{MeV}}{228} = 7.64 \frac{\text{MeV}}{\text{nucleon}}.$$

For thorium-232:

$$NBE = [90(1.00727u) + 142(1.00867u) - 231.98864u]c^2 \times \frac{931.5\text{MeV}}{1uc^2} = 1766.9\text{MeV}.$$

$$\text{The } \frac{NBE}{\text{nucleon}} = \frac{1766.9\text{MeV}}{232} = 7.62 \frac{\text{MeV}}{\text{nucleon}}.$$

8. What is the activity of 1 gram of radium-226?

The activity is given as the product λN , where λ is the decay constant and N is the number of nuclei that decay. Given that our sample is radium-226, with a half-life of 1600 years, we can

$$\text{calculate the decay constant. } \lambda = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{1600\text{yrs}} \times \frac{1\text{yr}}{3.2 \times 10^7\text{sec}} = 1.35 \times 10^{-11}\text{sec}^{-1}.$$

To calculate the number of nuclei present we use the mass given, 1g. There are 226 g of radium per mole and in 1 mole there are 6.02×10^{23} nuclei. Thus in 1 g there are 2.66×10^{21} nuclei. The activity is therefore $\lambda N = (1.35 \times 10^{-11}\text{sec}^{-1})(2.66 \times 10^{21}\text{ nuclei}) = 3.6 \times 10^{10}$ decays/sec = 3.6×10^{10} Bq.

9. Strontium is chemically similar to calcium and can replace calcium in bones. The radiation from ^{90}Sr can damage bone marrow where blood cells are produced, and lead to serious health problems. How long would it take for all but 0.01% of a sample of ^{90}Sr to decay?

Here we take the initial mass $M_0 = 1$ and the final mass is 0.01% of the initial mass. Thus $M_f = 1 \times 10^{-4} M_0$. The decay constant for ^{90}Sr is $\lambda = 0.024\text{yr}^{-1}$. From the radioactive decay law:

$$M_f = M_0 e^{-\lambda t} \rightarrow 1 \times 10^{-4} M_0 = M_0 e^{-(0.024\text{yr}^{-1})t} \rightarrow t = 383.4\text{yrs}.$$

10. After the sudden release of radioactivity from the Chernobyl nuclear reactor accident in 1986, the radioactivity of milk in Poland rose to 2 000 Bq/L due to iodine-131 present in the grass eaten by dairy cattle. Radioactive iodine, with half-life 8.04 days, is particularly hazardous because the thyroid gland concentrates iodine. The Chernobyl accident caused a measurable increase in thyroid cancers among children in Belarus. (a) For comparison, find the activity of milk due to potassium. Assume that one liter of milk contains 2.00 g of potassium, of which 0.011 7% is the isotope ^{40}K with half-life 1.28×10^9 yr. (b) After what time interval would the activity due to iodine fall below that due to potassium?

(a) One liter of milk contains this many ^{40}K nuclei:

$$N = (2.00 \text{ g}) \left(\frac{6.02 \times 10^{23} \text{ nuclei/mol}}{39.1 \text{ g/mol}} \right) \left(\frac{0.0117}{100} \right) = 3.60 \times 10^{18} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28 \times 10^9 \text{ yr}} \left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}} \right) = 1.72 \times 10^{-17} \text{ s}^{-1}$$

$$R = \lambda N = (1.72 \times 10^{-17} \text{ s}^{-1}) (3.60 \times 10^{18}) = \boxed{61.8 \text{ Bq}}$$

(b) For the iodine, $R = R_0 e^{-\lambda t}$ with $\lambda = \frac{\ln 2}{8.04 \text{ d}}$

$$t = \frac{1}{\lambda} \ln \left(\frac{R_0}{R} \right) = \frac{8.04 \text{ d}}{\ln 2} \ln \left(\frac{2000}{61.8} \right) = \boxed{40.3 \text{ d}}$$

11. A small building has become accidentally contaminated with radioactivity. The longest-lived material in the building is strontium-90. ($^{90}_{38}\text{Sr}$ has an atomic mass 89.907 7 u, and its half-life is 29.1 yr. It is particularly dangerous because it substitutes for calcium in bones.) Assume that the building initially contained 5.00 kg of this substance uniformly distributed throughout the building and that the safe level is defined as less than 10.0 decays/min (to be small in comparison to background radiation). How long will the building be unsafe?

$$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00 \text{ kg}}{(89.9077 \text{ u}) (1.66 \times 10^{-27} \text{ kg/u})} = 3.35 \times 10^{25} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1 \text{ yr}} = 2.38 \times 10^{-2} \text{ yr}^{-1} = 4.53 \times 10^{-8} \text{ min}^{-1}$$

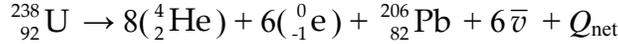
$$R_0 = \lambda N_0 = (4.53 \times 10^{-8} \text{ min}^{-1}) (3.35 \times 10^{25}) = 1.52 \times 10^{18} \text{ counts/min}$$

$$\frac{R}{R_0} = e^{-\lambda t} = \frac{10.0 \text{ counts/min}}{1.52 \times 10^{18} \text{ counts/min}} = 6.59 \times 10^{-18}$$

and $\lambda t = -\ln(6.59 \times 10^{-18}) = 39.6$

$$\text{giving } t = \frac{39.6}{\lambda} = \frac{39.6}{2.38 \times 10^{-2} \text{ yr}^{-1}} = \boxed{1.66 \times 10^3 \text{ yr}}.$$

12. Natural uranium must be processed to produce uranium enriched in ^{235}U for bombs and power plants. The processing yields a large quantity of nearly pure ^{238}U as a by-product, called “depleted uranium.” Because of its high mass density, it is used in armor-piercing artillery shells. (a) Find the edge dimension of a 70.0-kg cube of ^{238}U . The density of uranium is $18.7 \times 10^3 \text{ kg/m}^3$. (b) The isotope ^{238}U has a long half-life of $4.47 \times 10^9 \text{ yr}$. As soon as one nucleus decays, it begins a relatively rapid series of 14 steps that together constitute the net reaction



Find the net decay energy. (Refer to Table A.3.) (c) Argue that a radioactive sample with decay rate R and decay energy Q has power output $\wp = QR$. (d) Consider an artillery shell with a jacket of 70.0 kg of ${}^{238}\text{U}$. Find its power output due to the radioactivity of the uranium and its daughters. Assume that the shell is old enough that the daughters have reached steady-state amounts. Express the power in joules per year. (e) A 17-year-old soldier of mass 70.0 kg works in an arsenal where many such artillery shells are stored. Assume that his radiation exposure is limited to absorbing 45.5 mJ per year per kilogram of body mass. Find the net rate at which he can absorb energy of radiation, in joules per year.

(a) $V = \ell^3 = \frac{m}{\rho}$, so $\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0 \text{ kg}}{18.7 \times 10^3 \text{ kg/m}^3}\right)^{1/3} = \boxed{0.155 \text{ m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes
 ${}_{92}^{238}\text{U} \text{ atom} \rightarrow 8 {}_2^4\text{He} \text{ atom} + {}_{82}^{206}\text{Pb} \text{ atom} + Q_{\text{net}}$.

$$Q_{\text{net}} = \left[M_{{}_{92}^{238}\text{U}} - 8 M_{{}_2^4\text{He}} - M_{{}_{82}^{206}\text{Pb}} \right] c^2 = [238.050783 - 8(4.002603) - 205.974449] \text{u} (931.5 \text{ MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7 \text{ MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q . Then the energy released per time is $\boxed{P = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00 \times 10^4 \text{ g}}{238 \text{ g/mol}}\right) (6.02 \times 10^{23} \text{ nuclei/mol}) = 1.77 \times 10^{26} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} = 1.55 \times 10^{-10} \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55 \times 10^{-10} \frac{1}{\text{yr}}\right) (1.77 \times 10^{26} \text{ nuclei}) = 2.75 \times 10^{16} \text{ decays/yr},$$

$$\text{so } P = QR = (51.7 \text{ MeV}) \left(2.75 \times 10^{16} \frac{1}{\text{yr}}\right) (1.60 \times 10^{-13} \text{ J/MeV}) = \boxed{2.27 \times 10^5 \text{ J/yr}}.$$

(e) The allowed whole-body dose is then $(70.0 \text{ kg}) \left(\frac{4.55 \times 10^{-3} \text{ J}}{\text{kg} \cdot \text{yr}}\right) = \boxed{3.18 \text{ J/yr}}$.

13. A sealed capsule containing the radiopharmaceutical phosphorus-32 ($^{32}_{15}\text{P}$), an e^- emitter, is implanted into a patient's tumor. The average kinetic energy of the beta particles is 700 keV. The initial activity is 5.22 MBq. Determine the energy absorbed during a 10.0-day period. Assume that the beta particles are completely absorbed within the tumor.

The half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26 \text{ d}} = 0.0486 \text{ d}^{-1} = 5.63 \times 10^{-7} \text{ s}^{-1}.$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22 \times 10^6 \text{ decay/s}}{5.63 \times 10^{-7} \text{ s}^{-1}} = 9.28 \times 10^{12} \text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = (9.28 \times 10^{12} \text{ nuclei}) e^{-(0.0486 \text{ d}^{-1})(10.0 \text{ d})} = 5.71 \times 10^{12} \text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E = (3.57 \times 10^{12})(700 \text{ keV})(1.60 \times 10^{-16} \text{ J/keV}) = \boxed{0.400 \text{ J}}.$$

14. To destroy a cancerous tumor, a dose of gamma radiation totaling an energy of 2.12 J is to be delivered in 30.0 days from implanted sealed capsules containing palladium-103. Assume that this isotope has half-life 17.0 d and emits gamma rays of energy 21.0 keV, which are entirely absorbed within the tumor. (a) Find the initial activity of the set of capsules. (b) Find the total mass of radioactive palladium that these "seeds" should contain.

The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{17 \text{ d}} = 0.0408/\text{d}$. The number of nuclei remaining after 30 d

is $N = N_0 e^{-\lambda t} = N_0 e^{-0.0408(30)} = 0.294 N_0$. The number decayed is

$N_0 - N = N_0(1 - 0.294) = 0.706 N_0$. Then the energy release is

$$2.12 \text{ J} = 0.706 N_0 (21.0 \times 10^3 \text{ eV}) \left(\frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right)$$

$$N_0 = \frac{2.12 \text{ J}}{2.37 \times 10^{-15} \text{ J}} = 8.94 \times 10^{14}$$

$$(a) \quad R_0 = \lambda N_0 = \frac{0.0408}{\text{d}} (8.94 \times 10^{14}) \left(\frac{1 \text{ d}}{86400 \text{ s}} \right) = \boxed{4.22 \times 10^8 \text{ Bq}}$$

$$(b) \quad \text{original sample mass} = m = N_{\text{original}} m_{\text{one atom}} = 8.94 \times 10^{14} (103 \text{ u}) \left(\frac{1.66 \times 10^{-27} \text{ kg}}{1 \text{ u}} \right) \\ = \boxed{1.53 \times 10^{-10} \text{ kg}} = 1.53 \times 10^{-7} \text{ g} = 153 \text{ ng}$$

15. A living specimen in equilibrium with the atmosphere contains one atom of ^{14}C (half-life = 5730 yr) for every 7.7×10^{11} stable carbon atoms. An archeological sample of wood (cellulose, $\text{C}_{12}\text{H}_{22}\text{O}_{11}$) contains 21.0 mg of carbon. When the sample is placed inside a shielded beta counter with 88.0% counting efficiency, 837 counts are accumulated in one week. Assuming that the cosmic-ray flux and the Earth's atmosphere have not changed appreciably since the sample was formed, find the age of the sample.

$$N_C = \left(\frac{0.0210 \text{ g}}{12.0 \text{ g/mol}} \right) (6.02 \times 10^{23} \text{ molecules/mol})$$

($N_C = 1.05 \times 10^{21}$ carbon atoms) of which 1 in 7.70×10^{11} is a ^{14}C atom

$$(N_0)_{\text{C-14}} = 1.37 \times 10^9,$$

$$\lambda_{\text{C-14}} = \frac{\ln 2}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1} = 3.83 \times 10^{-12} \text{ s}^{-1}$$

$$R = \lambda N = \lambda N_0 e^{-\lambda t}$$

At $t = 0$,

$$R_0 = \lambda N_0 = (3.83 \times 10^{-12} \text{ s}^{-1})(1.37 \times 10^9) \left[\frac{7(86400 \text{ s})}{1 \text{ week}} \right] = 3.17 \times 10^3 \text{ decays/week}.$$

$$\text{At time } t, \quad R = \frac{837}{0.88} = 951 \text{ decays/week}.$$

$$\text{Taking logarithms,} \quad \ln \frac{R}{R_0} = -\lambda t \quad \text{so} \quad t = \frac{-1}{\lambda} \ln \left(\frac{R}{R_0} \right)$$

$$t = \frac{-1}{1.21 \times 10^{-4} \text{ yr}^{-1}} \ln \left(\frac{951}{3.17 \times 10^3} \right) = \boxed{9.96 \times 10^3 \text{ yr}}.$$

16. In an experiment on the transport of nutrients in the root structure of a plant, two radioactive nuclides X and Y are used. Initially 2.50 times more nuclei of type X are present than of type Y. Just three days later there are 4.20 times more nuclei of type X than of type Y. Isotope Y has a half-life of 1.60 d. What is the half-life of isotope X?

$$\text{We have all this information: } N_x(0) = 2.50N_y(0)$$

$$N_x(3\text{d}) = 4.20N_y(3\text{d})$$

$$N_x(0)e^{-\lambda_x 3\text{d}} = 4.20N_y(0)e^{-\lambda_y 3\text{d}} = 4.20 \frac{N_x(0)}{2.50} e^{-\lambda_y 3\text{d}}$$

$$e^{3\text{d}\lambda_x} = \frac{2.5}{4.2} e^{3\text{d}\lambda_y}$$

$$3\text{d}\lambda_x = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d}\lambda_y$$

$$3\text{d} \frac{0.693}{T_{1/2x}} = \ln\left(\frac{2.5}{4.2}\right) + 3\text{d} \frac{0.693}{1.60 \text{ d}} = 0.781$$

$$T_{1/2x} = \boxed{2.66 \text{ d}}$$