Relativistic Electrons

Physics 300

1 Introduction

In this experiment you will make independent measurements of the momentum and kinetic energy of electrons emitted from a β source. You will use these data to investigate the relationship between the momentum and kinetic energy, and to extract values for the speed of light and the rest mass of the electron.

The theory behind this experiment is based on the fact that an electron traveling with a velocity $\overrightarrow{v}$ perpendicular to a uniform magnetic field $\overrightarrow{B}$ experiences a force in a direction perpendicular to both the velocity and the magnetic field and with a magnitude given by

$$F = evB \tag{1}$$

where e is the magnitude of the charge of the electron. As shown in Figure 1, this force will cause the electron to travel in a circular path. Applying Newton's second law of motion we can express the magnitude of the momentum of the electron as

$$p = eBr \tag{2}$$

where r is the radius of the circular path. Thus the momentum of the electron can be determined by measuring the strength of the magnetic field and the radius of the circular path.

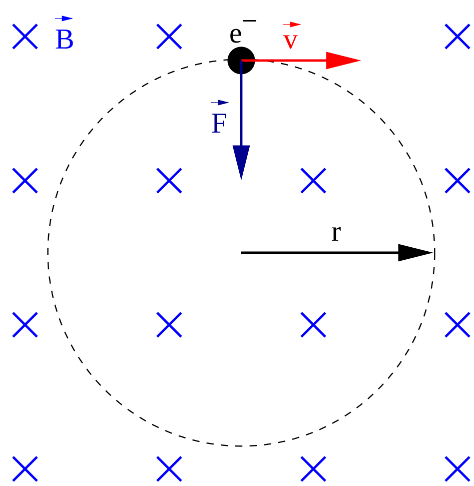

Figure 1: The path of an electron moving with a velocity perpendicular to a uniform magnetic field.

The relativistic expression for the kinetic energy K of the electron is

$$K = \sqrt{(pc)^2 + (mc^2)^2} - mc^2 \tag{3}$$

where m is the rest mass of the electron and c is the speed of light. In the classical limit this reduces to

$$K = \frac{p^2}{2m}. \tag{4}$$

Notice that equations 3 and 4 predict different relationships between the kinetic energy and the momentum, and therefore independent measurements of the kinetic energy and momentum over a range of electron energies will distinguish between the two expressions.

Rewriting the expression for the relativistic kinetic energy (equation 3) in the form of the equation of a line we have

$$\frac{p^2}{2K} = \left(\frac{1}{c^2}\right)\left(\frac{K}{2}\right) + m. \tag{5}$$

This result shows that a graph of $\frac{p^2}{2K}$ versus $\frac{K}{2}$ yields a line with a slope of $\frac{1}{c^2}$ and an intercept of m. Thus values for the speed of light and the rest mass of the electron can be extracted by fitting a line to the data.

2 Pre-lab Exercises

Do the following pre-lab exercises and hand them in separately from your lab report:

1. Derive equation 2.
2. Show that equation 3 reduces to 4 in the classical limit. (Hint: Use a binomial expansion.)
3. Create a graph of kinetic energy (in keV) as a function of momentum (in keV/c) showing both the relativistic and classical relationships over the momentum range of 0-1000 keV/c.
4. Derive equation 5.

3 Experimental Apparatus

A schematic diagram of the setup for this experiment is shown in Figure 2. The experiment will be conducted in a vacuum chamber positioned between the poles of an electromagnet. The vacuum chamber will be evacuated to a pressure of about 50 mTorr with a mechanical pump. Inside the chamber is a source, a detector, and three slits. The source of electrons will be a 10-μCi, ^{204}Tl beta source which emits a continuous spectrum of electrons with an end-point energy of 766 keV. The slits define a narrow beam of electrons that will travel in a circular path with a radius r. The kinetic energy of the electrons will be measured with the detector and the momentum will be determined by measuring the magnitude of the magnetic field with a Hall probe.

The detector that will be used in this experiment is a silicon surface-barrier detector. When a charged particle is stopped in the detector it produces ionization whose total charge is proportional to the energy of the particle. The ionized charge is collected on an electrode due to a bias potential across the detector and produces a current pulse. The current pulse is integrated by a pre-amplifier to produce a voltage pulse with an amplitude that is proportional to the energy of the particle.

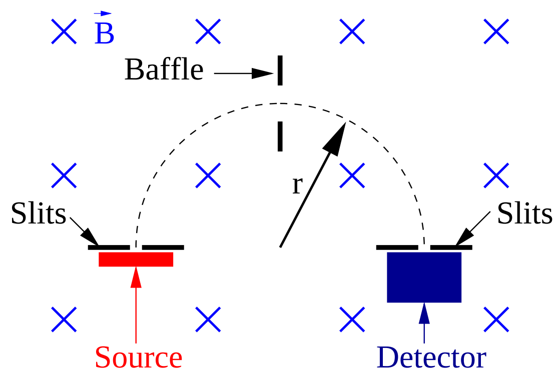

Figure 2: A schematic diagram of the experimental apparatus.

The amplitude of this voltage pulse is then increased with a linear amplifier and fed into a multi-channel analyzer to perform a pulse-height analysis. The energy spectrum will be calibrated using the 624-keV electron conversion peak from a ^{137}Cs source.

4 Experimental Procedure

1. Disconnect the vacuum line and the detector cable, and remove the vacuum chamber from between the poles of the electromagnet. Be sure to support the vacuum chamber as the bolts that go through the magnet and into the end-plates of the chamber are removed. Also note the orientation of the chamber before it is removed.

2. Place the chamber on the work bench so that the right end-plate is facing up. Remove the right end-plate and replace the slit in front of the detector with the slit that has the ^{137}Cs calibration source taped to it. Make sure that the O-ring is properly seated in the groove and screw the end-plate back onto the chamber.

3. Put the chamber back between the poles of the magnet and secure it by screwing the bolts through the magnet into the end-plates of the chamber. Connect the vacuum line and the detector cable. Then turn on the mechanical pump. In a few minutes the pump should settle down and make a consistently quiet pumping sound.

4. Turn on the NIM crate and the power supply, and set the detector bias to 550 V.

5. Launch the Maestro MCA Emulator software on the computer. Clear the spectrum by clicking on the Clear Spectrum icon on the toolbar or selecting Clear from the Acquire menu. Also, select Destroy Calibration from the Calculate/Calibration menu. Then start the data acquisition by clicking on the Start Acquisition icon on the toolbar or selecting Start from the Acquire menu. You should see events begin to accumulate in the spectrum.

6. When you have a well-defined electron conversion peak, stop collecting data. Click on the center of the peak and then on the Mark ROI (region of interest) icon on the toolbar. Select Calibration from the Calculate menu, enter 624 in the Calibration Energy window, and click OK. The spectrum is now calibrated and the peak should be centered at 624 keV. Click the Clear ROI button on the toolbar and save the spectrum in the .SPC format.

7. Turn down the detector bias and turn off the power supply and the mechanical pump. Disconnect the vacuum line and the detector cable, and remove the vacuum chamber from between the poles of the electromagnet. Open the chamber and replace the slit that has the ^{137}Cs calibration source taped to it with the empty slit. Use a vernier caliper to measure the width of the slits in front of the source and detector and the distance between the slits. Be careful not to touch the surface of the detector with the verier caliper. Make sure that the O-ring is properly seated in the groove and screw the end-plate back onto the chamber.

8. Put the chamber back between the poles of the magnet and secure it by screwing the bolts through the magnet into the end-plates of the chamber. Connect the vacuum line and the detector cable. Then turn on the mechanical pump. In a few minutes the pump should settle down and make a consistently quiet pumping sound. Then turn on the power supply and set the detector bias to 550 V.

9. Turn on the gauss meter. Remove the plastic cover and insert the probe into the zero-gauss chamber. Then press the ZERO button twice. A short time later the display should read around 0.000 KG. Insert the probe into the small hole in the wall of the vacuum chamber and orient the probe so that the plane of the probe is parallel to the pole faces of the magnet (so that the plane of the probe is perpendicular to the magnetic field). Please be gentle with the probe because it is delicate and can be damaged. Turn on the magnet power supply and slowly increase the current until the gauss meter reads around 500 gauss. Rotate the probe slowly and record the maximum value of the gauss meter reading. Then remove the probe from the hole in the vacuum chamber, put the plastic cover on the probe, turn off the gauss meter, and put it back on the workbench.

10. Clear the spectrum and start collecting data. We will collect data for anywhere between five and ten hours, depending on the magnetic field setting.

11. When the run is finished, stop the data acquisition and save the spectrum in .SPC format. Follow the procedure in step 9 to check the magnetic field reading and record the value. (Do not adjust the power supply when you make this reading.) Then follow the procedure in step 9 to increase the magnetic field by approximately 100 gauss. Clear the spectrum and start collecting data.

12. Repeat step 11 up to a magnetic field of about 1000 gauss.

5 Data Analysis

1. For each of your electron energy spectra, use the Maestro software to determine the centroid in keV, the full-width-at-half-maximum (FWHM) in keV, and the net area (number of electrons) of the peak. To do this, choose Recall from the File menu and select the spectrum of interest. Click on the center of the peak and expand the spectrum around the peak by hitting the "+" key multiple times. Select the area around the peak with the left mouse button and then click the right mouse button and select Mark ROI (region of interest). Then click the right mouse button and select Peak Info.

2. Calculate the electron momentum for each run from the measured values of the magnetic field and the radius of the path defined by the slits.

3. Plot the measured kinetic energy in keV as a function of momentum in keV/c for each run and compare with the predictions of equations 3 and 4. Be sure to include uncertainties on the data points.

4. Create a graph of $\frac{p^2}{2K}$ versus $\frac{K}{2}$ and fit the data to determine values for the speed of light and the rest mass of the electron. Be sure to include the uncertainties when you report these values.

6 References

For more information see the following references:

1. Couch and Dorries, "Measuring relativistic electrons in the undergraduate laboratory," Am. J. Phys. 50(10), 917, Oct. 1982.

2. Geller and Kollarits, "Experiment to Measure the Increase in Electron Mass with Velocity," Am. J. Phys. 40, 1125, August 1972.

3. Leo, "Techniques for Nuclear and Particle Physics Experiments," Second Edition, Springer-Verlag (1994).

4. Knoll, "Radiation Detection and Measurement," Second Edition, Wiley (1989).

5. Bevington and Robinson, "Data Reduction and Error Analysis for the Physical Sciences," Third Edition, McGraw-Hill (2003).