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CONTEMPORARY PROBLEMS IN MACROECONOMICS

The Solow Growth Model

- ***Version 1: No population growth, no technological progress***

Consider the following production function in which Y is real GDP, K is the capital stock, and N is the labor force:

$$Y = F(K, N). \tag{1.1}$$

Assume that the above is a constant-returns-to-scale production function. We can, therefore, write (1.1) as:

$$Y/N = F(K/N, 1). \tag{1.2}$$

Let $y \equiv Y/N$, and $k \equiv K/N$. We can then re-write (1.2) as:

$$y = f(k). \tag{1.3}$$

Assume that this is a closed economy with no government. Now, demand for goods and services in this economy can be written as:

$$y = c + i. \tag{1.4}$$

Where c and i are consumption per worker and investment per worker, respectively. In the Solow model consumers save a fraction s of their income. Therefore,

$$c = (1-s)y. \tag{1.5}$$

Consequently, from (1.5), (1.4) can be written as

$$y = (1-s)y + i. \tag{1.6}$$

Now, by a simple re-arrangement of terms in (1.6), we get:

$$i = sy. \tag{1.7}$$

Now, assume that the depreciation rate of capital is δ. Keeping in mind that sy in (1.7) can be written equivalently as *sf(k)*, the change in the capital stock (per worker) can then be written as:

$$\Delta k = sf(k) - \delta k. \tag{1.8}$$

The steady-state value of *k*, denoted by *k**, is given by (1.8), when Δk is set to zero.

The Golden Rule level of capital

The steady-state value of *k* that maximizes *c* is called the *Golden Rule* level of capital. Policymakers may be able to choose *s* such that, in steady state, *c* is maximized. Now, note that in steady state

$$c^* = f(k^*) - \delta k^*. \tag{1.9}$$

A necessary condition for maximum *c** is

$$f'(k^*) - \delta = 0 \tag{1.10}$$

So, to find the *Golden Rule saving rate*, the following two equations, derived from (1.8) and (1.10), must be solved:

$$sf(k^*) = \delta k \quad and \quad f'(k^*) = \delta. \tag{1.11}$$

Some highlights of the Solow model:

1. At the steady state, the (per capita GDP) growth rate is zero.
2. When $k<k^*$ positive growth takes place (and vice versa).
3. An increase in *s* will lead to short-term positive growth. The steady-state growth will be zero but at a higher *y*.

- ***Version 2: Positive population growth, no technological progress***

Now, let population and labor force grow at a constant rate g_N. Equation (1.8) must now be rewritten as

$$\Delta k = sf(k) - (\delta + g_N)k. \tag{2.1}$$

The steady-state condition is

$$sf(k) - (\delta + g_N)k = 0. \tag{2.2}$$

And the *Golden Rule saving rate* can be derived by solving (2.2) and

$$dc^*/dk = d[f(k^*) - (\delta + g_N)k^*]/dk = 0 \quad \Rightarrow \quad f'(k^*) = (\delta + g_N). \tag{2.3}$$

An additional insight in this version of the Solow model is that, all else equal, a country with a high rate of population growth will have a low steady state *k*, and therefore a low level of *y*.

- ***Version 3: Positive population growth, positive technological progress***

Re-write the production function in (1.1) as

$$Y = F(K, A \times N). \tag{3.1}$$

Where *A* is the efficiency of labor, and the term $A \times N$ is the labor force measured in efficiency units. Assume that technological progress causes *A* to grow at the rate of g_A. In this model the number of efficiency units of labor is growing at the rate $g_A + g_N$. Now, let

$y = Y/(A \times N)$ stand for output per efficiency unit of labor. The production function, as before, can be written as *y = f(k)*. Equation (1.8), previously modified to (2.1), must now be written as

$$\Delta k = sf(k) - (\delta + g_A + g_N)k. \tag{3.2}$$

The steady-state condition is

$$sf(k) - (\delta + g_A + g_N)k = 0. \tag{3.3}$$

And the *Golden Rule saving rate* can be obtained by solving (3.3) and

$$dc^*/dk = d[f(k^*) - (\delta + g_A + g_N)k^*]/dk = 0 \quad \Rightarrow \quad f'(k^*) = \delta + g_A + g_N. \tag{3.4}$$

Note that in this version of the Solow model *y* grows at the rate of g_A in the steady state.