R&D Policies, Endogenous Growth and Scale Effects*

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Abstract

This paper constructs a scale-free endogenous growth model and studies the determinants of optimal R&D policy. The model combines two of the main approaches to removal of scale effects: the rent protection approach and the diminishing technological opportunities approach. The steady-state rate of innovation is a function of all of the model's parameters including the R&D subsidy/tax rate. Thus, growth is fully endogenous. Numerical simulations imply that it is optimal to tax R&D when innovations are of very small and very large magnitudes, and to subsidize R&D when innovations are of medium size. Under a wide range of empirically relevant calibrations, the subsidy rate turns out to be positive and fluctuates between 5 to 25 percent.

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1. INTRODUCTION

Endogenous growth theory came at a crossroads with the Jones critique in the mid 1990s. First generation endogenous growth models predicted that the long-run growth rate of an economy increases in the level of R&D inputs and thus larger economies should grow at higher rates.[1] In two influential papers Jones (1995a, b) refuted this scale effect prediction by examining the post-war time series data from industrialized countries. In response, a second generation of endogenous growth models has emerged. This literature offers three main approaches to remove scale effects: *i)* Diminishing Technological Opportunities (henceforth **DTOs**) put forward by Jones (1995b), Kortum (1997) and Segerstrom (1998); *ii)* Rent Protection Activities (henceforth **RPAs**) proposed by Dinopoulos and Syropoulos (2007), *iii)* Variety Expansion (henceforth **VE**) proposed by Aghion and Howitt (1998, ch. 12), Dinopoulos and Thompson (1998), Howitt (1999), Peretto (1998) and Young (1998).[2]

In all of the above papers, growth is endogenous in the sense that it is driven by the innovation efforts of profit maximizing entrepreneurs. However the determinants of the steady-state growth rates differ markedly across these approaches. Models based on the DTO approach imply that the steady-state growth rate is exclusively pinned down by the rate of population growth and the rate of exhaustion in technological opportunities, leaving no room for R&D policies to exert an influence. Therefore, these models are often referred to as *semi-endogenous* growth models. In contrast, models using the RPA or VE approach predict that the steady-state growth rate is a function of *all* of the model's parameters including the R&D subsidy/tax rate. Thus, these models are often referred to as *fully-endogenous* growth models.

These stark differences in terms of steady-state outcomes are important not only in their own right but also because of their welfare implications. In a typical endogenous growth model, the search for welfare-maximizing optimal R&D policy involves the comparison of positive and negative externalities associated with a marginal unit of innovation. In the DTO based models, with only a small subset of parameters determining the rate of innovation, the majority of the parameters have no influence on the magnitudes of innovation externalities via their effect on the innovation rate. In contrast, in the RPA or VE based models, the entire set of the parameters do exert an influence through this particular channel. To see the implications for optimal R&D policy, compare for instance

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[1] See Romer (1990), Segerstrom, Anant and Dinopoulos (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992).

[2] See Dinopoulos and Şener (2007) for a recent analysis of scale-invariant growth theory. Jones (2005), Dinopoulos and Thompson (1999), and Jones (1999) also provide comprehensive overviews of the scale-invariant endogenous growth literature.

the results from the semi-endogenous growth model of Segerstrom (1998) and fully-endogenous growth model of Dinopoulos and Syropoulos (2007). Segerstrom (1998) finds that for small-sized innovations either R&D taxes or subsidies are optimal, whereas for sufficiently large-sized innovations R&D taxes are welfare maximizing. Quite the contrary, Dinopoulos and Syropoulos (2007) find that R&D taxes are optimal for small and large sized innovations, and R&D subsidies are optimal only for medium-sized innovations. It is easy to find more papers in this literature with major differences in R&D policy recommendations.[3]

Motivated by the above considerations I intend combine the DTO and RPA approaches under a unified setting and explore the implications for steady-state growth and R&D policy. I restrict the focus of the paper to these two approaches in order to facilitate the paper's comparison with the literature. The DTO approach captures the essence of the semi-endogenous growth theory, whereas the RPA approach captures the essence of the fully-endogenous growth theory. Incorporating the VE approach can of course be a fruitful avenue, which for now is left for further research.

Such a unified model can shed light on several important questions central to endogenous growth theory. When we combine the elements that give rise to fully-endogenous growth with those that give rise to semi-endogenous growth, will growth be fully endogenous or semi endogenous? Does the model point to taxes or subsidies as the optimal R&D policy? How do the externalities associated with marginal innovation respond to changes in parameters? When one calibrates the model what is the nature and extent of the optimal R&D policy?

The model is based on a standard quality-ladders growth setting in the tradition of Grossman and Helpman (1991, ch. 4). The economy is characterized by a continuum of structurally-identical industries. Labor is the only factor of production, and there are two types of labor: general-purpose and specialized labor. General-purpose workers can be employed in either R&D or manufacturing, and specialized workers can only be employed in RPAs. In each industry, entrepreneurs participate in R&D races to innovate higher quality products. The winner of an R&D race establishes monopoly power as the sole manufacturer of the highest quality product in the industry. Further innovation in the industry implies the emergence of a new quality leader and hence the replacement of the incumbent firm. The replacement rate faced by the incumbent firms is equal to the rate of innovation ι, which is endogenously determined by the profit-maximizing decisions of entrepreneurs.

I allow for positive population growth and remove the scale effects by introducing R&D difficulty at the industry level. I model R&D difficulty as a stock variable whose evolution is governed

[3] See Segerstrom (2007) and Li (2001, 2003) for a comparative analysis of R&D policies implied by different endogenous growth models.

by two distinct forces. First, as in Dinopoulos and Syropoulos (2007), RPAs undertaken by the incumbent monopolist in a particular industry raise the level of R&D difficulty for outside entrepreneurs who target their innovation efforts at this industry. These are costly activities which require the employment of specialized workers such as lawyers and lobbyists. Second, as in Segerstrom (1998, p.1297), within each industry, "the most obvious ideas are discovered first, making it harder to find new ideas subsequently." This sets in motion the DTO mechanism by which current innovation efforts raise the level of R&D difficulty for the subsequent periods. [4]

The model commands a unique steady-state equilibrium in which the rate of innovation remains constant in the presence of positive population growth. Thus, steady-state growth is free of scale effects. The equilibrium rate of innovation responds to all of the model's parameters including the R&D subsidy rate. Numerical simulations show that changes in R&D policies can have a sizeable impact on the innovation rate. Hence, the model predicts *fully-endogenous growth*. This is the first central result of the paper.

Even though there is no consensus on whether fully-endogenous or semi-endogenous growth better captures the real world, recent empirical studies by Ha and Howitt (2007) and Zachariadis (2003, 2004) lend more support for the former versus the latter. Ha and Howitt (2007) find that the predictions of the fully-endogenous growth theory (in particular, that the growth rate is a function of the fraction of the resources allocated to R&D and is endogenous) are more consistent with the time-series patterns from advanced countries vis-à-vis the predictions of the semi-endogenous growth theory (in particular, that the growth rate must follow the growth rate in R&D inputs). [5] My paper provides an additional insight by showing that the empirical evidence that favors fully-endogenous growth over semi-endogenous growth does *not* necessarily imply a rejection of the foundations of semi-endogenous growth theory. More specifically, the present model shows that the DTO mechanism that is at the heart of semi-endogenous growth theory can indeed be compatible with fully-endogenous growth when R&D difficulty accumulation has an added component that operates through rent protection by incumbents.

[4] In the working paper version (Şener, 2007), I considered two additional forces that affect R&D difficulty. The first is inter-industry knowledge spillovers by which improvements in the aggregate quality of goods can lead to technology spillovers and thus reduce the level of R&D difficulty in a typical industry—in the spirit of Li (2003) and Kortum (1997). The second is inter-industry rent protection spillovers by which increased RPAs at the aggregate level can hinder the access of entrepreneurs to current technologies and thereby raise the level of R&D difficulty in a typical industry. The main results of the paper are robust to inclusion of such spillovers.

[5] Ha and Howitt (2007) also conduct a comparison of the cointegration relations implied by each theory and conclude that fully-endogenous growth theory outperforms the semi-endogenous growth theory. See this paper and the references therein for an overview of the competing perspectives on this issue.

At the steady-state equilibrium of the model, the replacement rate as used in the stock market valuation of the incumbent firm *effectively* increases from ι to $\iota[1 + \eta]$, where η is an elasticity term that measures in percentage terms the effectiveness of RPAs in deterring outside innovation. This elasticity is *endogenous* and responds to all of the model's parameters. Hence the model establishes a novel link between the model's parameters and the *effective replacement rate* $\iota[1 + \eta]$ faced by the incumbent firm. This is the second central result of the paper. In Segerstrom (1998), RPAs are not considered and thus $\eta = 0$; whereas in Dinopoulos and Syropoulos (2007), there are RPAs but no consideration of DTOs and their model implies $\eta = 1$.

To understand the implications for R&D policy, I solve the optimization problem of a social planner whose objective is to maximize social welfare. I find that a marginal innovation generates three competing effects on welfare: a positive *consumer-surplus* externality, a negative *business-stealing* externality and a negative *intertemporal R&D spillover* externality. In this setting, the model's parameters influence these externalities directly and also indirectly via their effects on the innovation rate ι and innovation-deterring elasticity η.

The forces at work can be best understood by comparing the model with the two most related models. In Dinopoulos and Syropoulos (2007) there is no consideration of DTOs; therefore, the intertemporal R&D spillover externality is absent. In Segerstrom (1998) all of the three externalities are present; however, the innovation rate is exclusively determined by the two parameters, namely, the population growth rate and the exhaustion rate in technological opportunities. Thus, in Segerstrom there is no room for the rest of the parameters to influence the welfare externalities through the innovation rate. Lastly, in both Segerstrom (1998) and Dinopoulos and Syropoulos (2007) η is fixed; thus, the parameters do not influence the externalities through their effect on η.

Using benchmark values from the U.S., I calibrate the model and calculate the magnitude of optimal R&D policy. The benchmark simulations, taken at face value, imply that it is optimal to subsidize R&D at a rate of 15 percent. I check the robustness of this result by considering high and low values for all parameters. When the size of innovations is kept within the interval [1.20, 1.35] (which holds the share of R&D employment below 7 percent) and each parameter is allowed to vary within a band that keeps the innovation rate within the range [0.01, 0.04], the optimal R&D subsidy rate remains between 5 and 25 percent.

Allowing for a wider range of innovation size [1.10, 2.75], I find that it is optimal to tax R&D when innovations are of very small and very large magnitudes, and it is optimal to subsidize R&D when innovations are of medium size. This "n-shaped" relationship is a robust feature of the model and is exclusively tied to the fully-endogenous growth nature of the model. This finding implies that combining the RPA approach of Dinopoulos and Syropoulos (2007) with the DTO approach of Segerstrom (1998) resurrects the n-shaped relationship, which was originally proposed by Grossman and Helpman (1991). I also consider the model with alternative labor assignment schemes across activities. The main results are robust to such considerations.

The rest of the paper is organized as follows. Section 2 introduces the building blocks of the model and establishes the steady-state equilibrium. Section 3 presents the comparative steady-state analysis. Section 4 discusses the optimal R&D policies. Section 5 presents the simulation results. Section 6 considers variants of the model. Section 7 concludes. Proofs of all propositions are relegated to Appendices, which are available on my website http://minerva.union.edu/senerm/.

2. THE MODEL

2.1. The household's utility maximization

The economy consists of a continuum of identical households with measure of one. The size of each household at time t is $N(t) = e^{nt}$, where the initial level of population is normalized to one and $n > 0$ denotes the population growth rate. Each household takes goods prices, wages, and the interest rate as given and maximizes the following utility function over an infinite horizon

$$U = \int_0^{\infty} e^{-(\rho - n)t} \log u(t)\, dt , \tag{1}$$

where ρ is the subjective discount rate, and $\log u(t)$ is the instantaneous utility of each household member defined as:

$$\log u(t) = \int_0^1 \log\left[\sum_j \lambda^{j(\omega,t)} x(j,\omega,t)\right] d\omega \tag{2}$$

where $x(j, \omega, t)$ is the quantity demanded of a product with quality j in industry ω at time t. The size of quality improvements is denoted by $\lambda > 1$. Therefore, the total quality of a good after j innovations is λ^j.

Each household allocates its per capita consumption expenditure $c(t)$ to maximize $u(t)$ given prices at time t. Equation (2) implies that within each industry, products of different quality are perfect substitutes; thus, in each industry households purchase only the product with the lowest quality-adjusted price. Since products enter the utility function symmetrically, households spread their consumption expenditure evenly across the continuum of product lines. Consequently, demand for each product line by a household member is $x(j, \omega, t) = c(t)/P_m$ where P_m is the market price for the product that has the lowest quality-adjusted price.

Given the static demand behavior, the household's dynamic problem is simplified to maximizing

$$\int_0^{\infty} e^{-(\rho-n)t} \log c(t)\, dt, \tag{3}$$

subject to the budget constraint $\dot{A}(t) = W(t) + r(t)A(t) - c(t)N(t)$, where $A(t)$ denotes the financial assets owned by the household, $W(t)$ is the family's expected wage income and $r(t)$ is the instantaneous rate of return.[6] The solution to this optimization gives the standard differential equation

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \rho\,. \tag{4}$$

2.2. Activities and Labor Assignment

Firms conduct three types of activities: innovation, manufacturing of final goods and rent-protection. Labor is the only factor of production and there are two types of labor: general-purpose and specialized labor. General-purpose workers can be employed in either manufacturing or innovation, whereas specialized workers can only be employed in RPAs.[7] The population share of general-purpose workers is $(1 - s)$ and that of specialized workers is s, where $s \in (0,1)$.

2.3. R&D Races

The economy consists of a continuum of structurally-identical industries indexed by $\omega \in (0,1)$. Entrepreneurs participate in industry-specific R&D races to innovate higher quality products. An R&D race in industry ω is aimed at improving the quality of the existing product by a fixed size $\lambda > 1$. The winner of an R&D race gains access to the technology of producing the next-generation product and establishes monopoly power in the product market. Further innovation in the industry results in the replacement of the incumbent firm by a new quality leader.

The arrival of innovations in each industry is governed by a stochastic Poisson process, whose intensity is determined by the profit maximizing decisions of entrepreneurs. Let $R_j(\omega, t)$ represent the innovation intensity of a typical entrepreneur indexed by j targeting industry ω. The instantaneous probability of innovation success by firm j is

$$\iota_j(\omega,t) = \frac{R_j(\omega,t)}{D(\omega,t)}, \tag{5}$$

[6] I assume that, intra-household transfers ensure that per-capita consumption expenditure is the same for all household members when individually earned wages may differ.

[7] I assume labor mobility between manufacturing and R&D to model the economy's ability to allocate resources in different activities over the long run. This is a standard assumption in the growth literature. I assume a constant share of specialized labor to capture the established institutional set up associated with rent protection activities. With specialized labor, I basically mean lawyers, lobbyists and other individuals who possess rent-protection-activity-specific expertise which is not applicable to manufacturing or R&D. This particular labor distribution follows Dinopoulos and Syropoulos (2007). In Section 6, I consider alternative labor assignment schemes and study the implications for steady-state and welfare.

where $D(\omega,t)$ measures the difficulty of conducting innovation in industry ω. The probability of innovation success is distributed independently across firms and industries. Thus, the instantaneous probability of innovation success at the industry level equals

$$\iota(\omega,t) = \sum_j \iota_j(\omega,t) = \frac{R(\omega,t)}{D(\omega,t)}, \tag{6}$$

where $R(\omega,t) = \Sigma_j R_j(\omega,t)$.

In each industry, the incumbent monopolist (i.e., the quality leader) hires specialized labor to deter the innovation efforts of outside entrepreneurs. Let $X(\omega,t)$ stand for the level of RPAs undertaken by the incumbent firm in industry ω. Summing up $X(\omega,t)$ and $\iota(\omega,t)$ across structurally-identical industries gives the aggregate level of RPAs as $X_A(t) = \int_0^1 X(\omega,t)d\omega$ and the aggregate rate of innovation as $\iota_A(t) = \int_0^1 \iota(\omega,t)d\omega$.

I model $D(\omega,t)$ as a stock variable with the following equation of motion: [8]

$$\dot{D}(\omega,t) = \delta X(\omega,t) + \mu\iota(\omega,t)D(\omega,t), \tag{7}$$

where $\delta > 0$ and $\mu > 0$ are exogenously given. Equation (7) implies that two distinct forces govern the evolution of $D(\omega,t)$ over time.[9] The first term $\delta X(\omega,t)$ captures the rent protection effect à la Dinopoulos and Syropoulos (2007). The RPAs undertaken by the monopolist firm operating in industry ω raise the stock of R&D difficulty faced by the entrepreneurs who target their R&D efforts at industry ω. Rent protection activities can involve excessive patenting activities, patent enforcement through litigation, practicing trade secrecy, lobbying the government to affect legislation, engaging in corrupt activities to influence the legal/political system, and so on.[10] The second term $\mu\iota(\omega,t)D(\omega,t)$

[8] This differs from Dinopoulos and Syropoulos (2007) who model R&D difficulty as a flow variable which depreciates 100 percent at each instant in time. I considered the stock formulation for three reasons. First, RPAs have effects that persist over time since they influence the legislative and judicial system. Second, the stock formulation can easily accommodate an exogenous rate of depreciation for R&D difficulty, *DEPR*, with $0 \leq DEPR < 1$. Thus it offers a more flexible specification compared to Dinopoulos and Syropoulos (2007). Third, the stock formulation provides a convenient template to combine the DTO and RPA approaches and also incorporate inter-industry knowledge and rent-protection spillovers [See Şener 2007, working paper]. The main results are robust to the inclusion of *DEPR* and spillovers. I thus omitted them to economize on the notation.

[9] With its additive form, equation (7) provides a generalized equation of motion for R&D difficulty. When $\delta = 0$ and $\mu > 0$, the formulation boils down to that of Segerstrom (1998). When $\mu = 0$ and $\delta > 0$, the formulation captures the RPA approach of Dinopoulos and Syropoulos (2007). To generate a positive level of $D(t)$ at the steady-state, the necessary and sufficient conditions are: *i)* the initial level of R&D difficulty at time zero $D_0(t)$ being strictly positive, *ii)* either δ or μ being strictly positive.

[10] For detailed empirical evidence on RPAs, see Dinopoulos and Syropoulos (2007), Şener (2006) and the references therein. See also Levin et al. (1987) and Cohen et al. (2000) for survey-based evidence from the US manufacturing industries. Some empirical evidence from the US can be briefly presented here. According to

captures the DTO effect à la Segerstrom (1998), Jones (1995) and Kortum (1997). In Segerstrom's words (1998, p. 1297) the idea is that "firms start off exploring the *ex ante* most promising projects and when success does not materialize, they gradually switch to *ex ante* less promising projects." Thus, current research efforts raise the stock of R&D difficulty for subsequent periods.

2.4. Product Markets

The quality leader in industry ω can produce a product that is λ times better than the current-generation product. Manufacturing one unit of final good requires one unit of general-purpose labor regardless of the quality level of the product. I normalize the wage rate of general-purpose labor $w_G(t)$ to one and hence the unit cost of production. In each industry, a quality leader competes against a follower who can produce the product that is one step down in the quality ladder. Firms compete in prices. The quality leader engages in limit pricing by charging $P = \lambda$ and forces the follower to exit the market. The leader's monopoly profits from product sales are:

$$\pi^P(\omega,t) = \frac{\lambda - 1}{\lambda} c(\omega,t) N(t) \qquad (8)$$

where $\lambda - 1$ is the profit margin per unit of product and $c(\omega,t)N(t)/\lambda$ is the total demand for product in industry ω. During its tenure, the quality leader hires specialized workers to deter the innovation efforts of its rivals. Let γ represent the unit labor requirement of such rent protection activities and let $w_S(t)$ represent the wage rate of specialized labor. The total cost of conducting $X(\omega,t)$ units of RPA is $\gamma w_S(t)X(\omega,t)$. With $w_G(t)$ normalized to one, the relative wage between specialized and general-purpose labor can be stated as $w(t) \equiv w_S(t)/w_G(t) = w_S(t)$. The quality leader's profit flow net of rent protection costs boils down to: [11]

$$\pi(\omega,t) = \frac{\lambda - 1}{\lambda} c(\omega,t) N(t) - w(t)\gamma X(\omega,t)\,. \qquad (9)$$

2.5. Stock Markets

There exists a stock market that channels the savings of consumers to firms. Consider the stock market valuation of a quality leader $\upsilon(\omega,t)$ operating industry ω at time t. Over a small time interval of dt, the stockholders of the quality leader receive $\pi(\omega,t)dt$ in the form of dividend payments.

AIPLA (1997) direct legal costs of patent litigation range between \$1.0 and \$3.0 million (in 1997 dollars) for each side through the trial. Lerner (1995, p. 470) reports that the costs of patent litigation cases started in 1991 will account for 27% of total R&D expenditures of US companies in that year. Time series analysis of Somaya (2002, Figures 3 and 5) suggests that patent litigation has been pervasive in all six broad industries as classified by the USPTO. In a survey of biotech firms Lerner (1995) finds that 55 percent of small firms and 33 percent of large firms cite litigation as a *deterrent* to innovation.

[11] As in the standard quality-ladders growth model, it is not profitable for the monopolist to undertake R&D in order to extend its lead over the followers [see for instance Grossman and Helpman (1991), p. 93].

During the same time period, with probability $\iota(\omega,t)dt$, an outside entrepreneur successfully innovates the next generation product. In this event, the stockholders face a capital loss in the amount of $\upsilon(\omega,t)$. With probability $1-\iota(\omega,t)dt$, no innovation takes place in the industry, and the stockholders realize an appreciation/depreciation in their holdings by $\dot{\upsilon}(\omega,t)$. In the absence of any arbitrage opportunities, the expected rate of return from holding stocks issued by the quality leader must be equal to the risk-free market rate of return $r(t)$. This implies:

$$\frac{\pi(\omega,t)}{\upsilon(\omega,t)}dt+(1-\iota(\omega,t)dt)\frac{\dot{\upsilon}(\omega,t)}{\upsilon(\omega,t)}dt+\frac{0-\upsilon(\omega,t)}{\upsilon(\omega,t)}\iota(\omega,t)dt=r(t)dt. \tag{10}$$

Taking limits as $dt\rightarrow 0$ yields:

$$\upsilon(\omega,t)=\frac{\pi(\omega,t)}{r(t)+\iota(\omega,t)-\dfrac{\dot{\upsilon}(\omega,t)}{\upsilon(\omega,t)}}. \tag{11}$$

2.6. Free-entry in R&D races

Entrepreneurs hire general-purpose labor to perform innovative activity and participate in R&D races. With w_G normalized to one, the cost of conducting $R_j(\omega,t)$ units of R&D for a typical entrepreneur indexed by j equals $a_\iota R_j(\omega,t)$, where a_ι is the unit labor requirement of R&D. The expected profits of an entrepreneur targeting its innovation efforts at industry ω are:

$$\upsilon(\omega,t)\frac{R_j(\omega,t)}{D(\omega,t)}dt-a_\iota(1-\phi_\iota)R_j(\omega,t)dt \tag{12}$$

where ϕ_ι is the R&D subsidy rate offered by the government. Over a time interval dt, the entrepreneur realizes a value of $\upsilon(\omega,t)$ with probability $\iota_j(\omega, t)= [R_j(\omega, t)/D(\omega, t)]dt$ and incurs a cost of $a_\iota(1-\phi_\iota)R_j(t)dt$. Free entry into R&D races drives the expected profits down to zero. This implies:

$$\frac{\upsilon(\omega,t)}{D(\omega,t)}=a_\iota(1-\phi_\iota). \tag{13}$$

2.7. Optimal rent protection decisions

The RPAs undertaken by the incumbent monopolists (i.e., quality leaders) prolong their monopoly power and thus raise the expected returns on their stocks. The incumbents choose the optimal level of $X(\omega,t)$ by equating the incremental gain in the expected return on their stocks to the incremental cost incurred to hire the additional specialized workers.

To obtain the associated first order condition, the first step involves deriving $d\iota(\omega,t)/dX(\omega,t)$, namely, the response of R&D intensity that targets industry ω, $\iota(\omega,t)$, to changes in the level of innovation-deterring activities undertaken by the incumbent in industry ω, $X(\omega,t)$. Consider an increase in the level of rent protection activity by dX, beginning at time t and extending over a small time interval dt. This increases the accumulation rate for $D(\omega,t)$ and lowers $\iota(\omega,t)$ via (6). Let $d\iota(\omega,t)$ measure the resulting change in $\iota(\omega,t)$ over the interval dt. To derive $d\iota(\omega,t)/ dX(\omega,t)$, I start by evaluating the difference in the R&D difficulty levels between time t and $t + dt$, $D(\omega, t + dt) - D(\omega, t)$, due to increased $X(\omega,t)$. Two effects are at work. One is the direct effect that operates through the rent-protection channel. The higher $X(\omega,t)$ increases the rate of accumulation for $D(\omega,t)$ over the interval dt, raising the R&D difficulty level at time $t + dt$. The other is the indirect effect that operates through the DTO channel. The reduction in $\iota(\omega,t)$ by $d\iota$ units over the interval dt decreases the rate of accumulation for $D(\omega,t)$, lowering the R&D difficulty level at time $t + dt$.

To capture the above effects, I need to define and evaluate two terms. The first is D_X, which measures the change in D due to X being increased by dX units over the interval dt. The second is D_ι, which measures the change in D due to ι being increased by $d\iota$ units over the interval dt. Taking limits as $d\iota \to 0$, $dX \to 0$, and $dt \to 0$, it immediately follows from (7) that:

$$D_X \equiv \frac{\partial\left(\frac{\partial D(\omega,t)}{\partial t}\right)}{\partial X} = \delta \text{ and } D_\iota \equiv \frac{\partial\left(\frac{\partial D(\omega,t)}{\partial t}\right)}{\partial \iota} = \mu D(\omega,t)\,. \tag{14}$$

Hence $dD(\omega) = D_X dX\, dt + D_\iota d\iota\, dt$, given the change in X as dX and the change in ι as $d\iota$ over the time interval dt. Totally differentiating $\iota(\omega,t)$ then implies:

$$dt d\iota = -\frac{R(\omega,t)}{D(\omega,t)^2}(D_X dX + D_\iota d\iota)dt\,. \tag{15}$$

Substituting for D_X and D_ι from (14), using (6), and taking limits as $dt \to 0$ gives:

$$\frac{d\iota}{dX} = -\frac{\delta R(\omega,t)}{D(\omega,t)^2[1+\mu(R(\omega,t)/D(\omega,t))]} = -\frac{\delta\, \iota(\omega,t)}{D(\omega,t)[1+\mu\iota(\omega,t)]}\,, \tag{16}$$

which provides an expression for $d\iota/dX$ and completes the first step of the analysis. Note for future use that according to (16), a diminishing returns relationship exists between $D(\omega,t)$ and $|d\iota/dX|$: as the stock of R&D difficulty increases, the effectiveness of RPAs declines. Moreover, (16) implies the following:

Lemma 1: *Ceteris paribus, the presence of DTOs as captured by $\mu\iota > 0$ reduces the marginal effectiveness of RPAs in deterring innovation (as measured by $|d\iota/dX|$). I call this "the DTO-RPA interaction mechanism."*

Intuitively, whenever an incumbent firm raises its RPAs and thus deters innovation, it indirectly mitigates the impact of DTOs on research difficulty. This is a novel mechanism that arises from the joint modeling of RPAs and DTOs.

The second step is to evaluate the change in the expected return on the incumbent's stocks due to the fall in $\iota(\omega,t)$ induced by an increase in $X(\omega,t)$ over a time interval dt in the amount dX. Differentiating $\pi(\omega,t)dt + [0 - \upsilon(\omega,t)]\,\iota(\omega,t)dt + \dot{\upsilon}\,[1 - \iota(\omega,t)dt]dt$ with respect to $\iota(\omega,t)$ yields the incremental gain in the expected return as:

$$-\upsilon(\omega,t)\frac{d\iota}{dX}dXdt - \dot{\upsilon}\frac{d\iota}{dX}dXdtdt\,. \tag{17}$$

At the optimal level of $X(\omega,t)$, this must equal the incremental expenditure on specialized labor $w(t)\gamma dXdt$ over a time interval dt. Imposing this condition and taking limits as $dt \to 0$ gives:

$$-\upsilon(\omega,t)\frac{d\iota}{dX} = w(t)\gamma\,. \tag{18}$$

Substituting for $d\iota/dX$ from (16) into (18) yields the first order condition for optimal $X(\omega,t)$:

$$\frac{\delta\iota(\omega,t)\upsilon(\omega,t)}{D(\omega,t)[1+\mu\iota(\omega,t)]} = w(t)\gamma\,. \tag{19}$$

2.8. Labor Markets

Demand for general-purpose labor comes from manufacturing and R&D. In each industry, entrepreneurs hire $R(\omega,t)a_\iota$ units of labor to conduct innovative activity, and the incumbent firm hires $c(t)N(t)/\lambda$ units of labor for manufacturing purposes. The economy-wide demand for general-purpose labor is $\int_0^1 \left[R(\omega,t)a_\iota + \left(c(t)N(t)/\lambda\right)\right]d\omega = R_A(t)a_\iota + (c(t)N(t)/\lambda)$. The equilibrium condition for the general-purpose labor market can then be stated as:

$$(1-s)N(t) = R_A(t)a_\iota + \frac{c(t)N(t)}{\lambda}\,. \tag{20}$$

Demand for specialized labor comes from RPAs. In each industry, incumbent firm hires $\gamma X(\omega,t)$ units of specialized labor to conduct such activities. The economy-wide demand for specialized labor is $\int_0^1 \gamma X(\omega,t)d\omega = \gamma X_A(t)$. The equilibrium condition for the specialized labor market then becomes:

$$sN(t) = \gamma X_A(t). \tag{21}$$

2.9. Steady-State Equilibrium

I now solve the model for a steady-state equilibrium in which all endogenous variables attain strictly positive values and the rate of innovation $\iota(t)$ remains constant over time. The stability of this equilibrium is shown in Appendix A. At the steady-state $c(t)$, $w(t)$ and $r(t)$ remain constant over time, and $X(t)$, $\upsilon(t)$, $D(t)$, and $\pi(t)$ grow at the rate of n. From this point on, I drop the time index for the variables that remain constant at the steady-state.

Given the structural symmetry and measure one of industries, it follows that $R_A(t)= R(\omega,t)= R(t)$, $\iota_A = \iota(\omega,t)= \iota$ and $X_A(\omega,t) = X(\omega,t) = X(t)$. To simplify notation, I henceforth drop the industry index ω as well. Imposing $\dot{D}/D = n$ on equation (7) and solving for $D(t)$ gives:

$$D(t) = \frac{\delta X(t)}{n-\iota\mu} \qquad \Rightarrow \qquad \frac{D(t)}{X(t)} = \frac{\delta}{n-\iota\mu}, \tag{22}$$

which implies that $D(t) > 0$ requires $\iota < n/\mu$. Let $\eta(\iota)$ be defined as:

$$\eta(\iota) \equiv -\frac{d\iota}{dX}\frac{X}{\iota}$$

where $\eta(\iota)$ represents *the innovation-deterring elasticity of RPAs*. Substituting for $d\iota/dX$ from (16) and $D(t)$ from (22) into the $\eta(\iota)$ expression above gives:

$$\eta(\iota) \equiv \frac{1}{(1+\mu\iota)}\frac{\delta X(\mathrm{t})}{D(t)} = \frac{n-\iota\mu}{(1+\mu\iota)}, \tag{23}$$

Lemma 2: *At the steady-state, the partial derivatives of the innovation-deterring elasticity $\eta(\iota)$ are as follows:*

- *$\partial\eta(\iota)/\partial\iota < 0$ because of two effects. First, a higher ι increases $D(t)/X(t)$ through the DTO channel via (22). Second, a higher ι triggers the "DTO-RPA interaction mechanism" as identified in Lemma 1. Both effects reduce the effectiveness of rent protection and hence $\eta(\iota)$.*
- *$\partial\eta(\iota)/\partial\mu < 0$ because of the same two effects identified above.*

Lemma 2 demonstrates the endogeneity of $\eta(\iota)$, which constitutes a major departure from the literature where $\eta(\iota)$ is modeled a rigid parameter. In Dinopoulos and Syropoulos (2007), $\eta(\iota) = 1$, and in Segerstrom (1998), $\eta(\iota) = 0$. [12] Observe that the joint modeling of DTOs and RPAs play a crucial role in rendering $\eta(\iota)$ endogenous.

[12] I should note that imposing $\mu = 0$ gives $\eta(\iota) = n$. This differs from Dinopoulos and Syropoulos (2007), where $\eta(\iota) =1$. In Dinopoulos and Syropoulos (2007), R&D difficulty is modeled as a flow variable and thus the effectiveness of innovation deterring $-d\iota/dX$ does not get discounted by getting multiplied by n when the steady-state level of $D(t)$ is substituted. In the present paper, I model R&D difficulty as a stock variable and assume that firms choose their optimal RPA levels by looking dt periods ahead. Thus, when $D(t)$ is substituted from (22) into

Substituting for $D(t)$ from (22) into (19) simplifies the first order condition for optimal $X(t)$ as:

$$w\gamma X(t) = \iota\eta(\iota)\upsilon(t). \tag{24}$$

Equation (24) has a straightforward interpretation. At the steady-state, the level of rent protection expenditures $w\gamma X(t)$ increases when the incumbent faces a larger threat of replacement (higher ι), when RPAs become more effective (higher $\eta(\iota)$), or when the incumbent has more capital loss at stake due to replacement (higher $\upsilon(t)$). It follows from Lemma 2 and (24) that one can further decompose the effects that operate through the $\eta(\iota)$ channel. Outside innovation efforts ($\iota > 0$) and DTOs ($\mu > 0$) have a restraining impact on RPA expenditures since they reduce $\eta(\iota)$. Intuitively, when either ι or μ is larger, the stock of R&D difficulty in a given industry attains a higher level on its own. This renders the incumbent firm's rent protection efforts less effective and thereby reduces $\eta(\iota)$. In addition, when ι or μ is larger, this triggers the RPA-DTO interaction mechanism identified in Lemma 1, which also works to reduce $\eta(\iota)$.

Substituting $\pi(t)$ from (9) into (11) using $w\gamma X(t)$ from (24) gives the stock market valuation of the firms as:

$$\upsilon(t) = \frac{[(\lambda - 1)/\lambda]cN(t)}{\rho - n + \iota[1 + \eta(\iota)]}. \tag{25}$$

In equation (25), the numerator is the incumbent's profit flow from product sales and the denominator is the adjusted discount rate which takes into account the replacement rate faced by the incumbent firm.

Lemma 3: *The effective replacement rate faced by a monopolist equals* $\iota[1 + \eta(\iota)]$.

Lemma 3 establishes a novel negative link between innovation-deterring elasticity $\eta(\iota)$ and the stock market valuation of quality leaders. Intuitively, any increase in η, say by $d\eta$ units, *holding all else constant*, raises the effectiveness of RPAs and induces the monopolist to increase its expenditure on rent protection by $\iota\upsilon(t)d\eta$ units *at each point in time* [via (24)]. This incremental expenditure flow leads to a fall in the firm's stock market valuation, which amounts to an increase in the effective replacement rate by $\iota d\eta$ units [via (25)].[13] Thus, shocks to μ as well as changes in the innovation rate ι have an additional impact on firm value through the $\eta(\iota)$ term.

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(16), the n term pops up as a coefficient of discount in front of $-d\iota/dX$. Under the stock formulation, if one assumes instead that incumbents have a perfect foresight of the steady-state equilibrium and choose their rent protection efforts $X(t)$ based on their steady-state impact, one obtains $\eta(\iota) = 1$ here as well. The main results are robust to this alternative behavioral assumption.

[13] Here one may wonder why firms undertake RPAs if this effectively increases their replacement rate from ι to $\iota[1 + \eta(\iota)]$. This is not a proper assessment though because ι is endogenous and its equilibrium levels may differ across models. In the scale-dependent endogenous growth models, ι increases with the population size $N(t)$.

I establish the steady-state equilibrium in (c, ι) space by obtaining two steady-state relationships: the competitive equilibrium free-entry in R&D condition, **RD^{CE}**, and the general-purpose labor market equilibrium condition **LM**. Substituting $\upsilon(t)$ from (25), $D(t)$ from (22), $\eta(\iota)$ from (23), and $X(t)$ from (21) into (13) gives RD^{CE}:

$$\frac{sA_\iota(1-\phi_\iota)}{n-\iota\mu}=\frac{[(\lambda-1)/\lambda]c}{\rho-n+\iota\left[1+\frac{(n-\iota\mu)}{(1+\mu\iota)}\right]}, \qquad \mathbf{RD^{CE}} \qquad (26)$$

where $A_\iota \equiv a_\iota\delta/\gamma$ is measure of resource requirement in R&D. Clearly, $\left(dc/d\iota\right)\big|_{RD^{CE}} > 0$. For a given c, an increase in ι exerts two main effects on R&D profitability. First, it increases R&D difficulty $D(t)$ through the DTO channel and renders R&D more costly. Second, an increase in ι increases the effective replacement rate $\iota[1+\eta(\iota)]$ despite the mitigating factor stemming from the fall in $\eta(\iota)$ (see Lemma 2). This reduces the rewards from R&D. Both effects work to decrease R&D profitability. To restore equilibrium, the rewards from R&D must increase through an increase in c.

I now derive the LM condition. Note that in (20) $R_A(t) = R(t) = \iota D(t)$ follows from (6) and structural symmetry. Substituting for $D(t)$ from (22) into (20) using $X(t) = sN(t)/\gamma$ from (21) gives:

$$1-s=\frac{A_\iota s}{(n-\iota\mu)}\iota+\frac{c}{\lambda}. \qquad \mathbf{LM} \qquad (27)$$

Clearly, $\left(dc/d\iota\right)\big|_{LM} < 0$. For a given c, an increase in ι directly raises the demand for R&D labor. At the same time, a higher ι raises $D(t)$ through the DTO channel, increasing the resource requirement in R&D. Both effects work to increase the demand for general-purpose labor. To restore labor market equilibrium, labor demand must fall through a decline in c.

I illustrate the steady-state equilibrium in Figure 1 with the intersection of the RD^{CE} and LM curves in (ι, c) space. Denote with "*" the steady-state equilibrium levels. It is straightforward to show that there exists a unique equilibrium for (ι^*, c^*) under the parametric condition: $(\lambda - 1)(1 - s)n >$

Thus, the replacement rate can increase without bound if there is population growth! In the present model, ι does not depend on $N(t)$, but on the growth rate of $N(t)$ (see Proposition 1). Hence, the replacement rate remains constant in the presence of population growth. On the other hand, it is possible to make a comparison with Segerstrom's (1998) scale-free growth model without RPAs, where the replacement rate is $\iota = n/\mu$. Using the expression for $\eta(\iota)$ from (23), it can be easily shown that $\iota[1 + \eta(\iota)] < n/\mu$ holds when the condition for an interior equilibrium with positive RPAs $\iota < n/\mu$ is satisfied [note that for $D > 0$, it follows from (22) that $n - \iota\mu < 0$ must hold]. Hence, the effective replacement rate with RPAs is lower than that of Segerstrom's. Intuitively, the presence of RPAs generates an additional factor that contributes to the accumulation of R&D difficulty and reduces the equilibrium innovation rate.

$sA_t(\rho - n)(1 - \phi_t)$.[14] One can then determine the equilibrium values for the rest of the endogenous variables in a recursive fashion. Substituting c^* into (8) gives $\pi_P^*(t)$. Substituting $X^*(t)$ from (21) and ι^* into (22) gives $D(t)^*$. Substituting ι^* and c^* into (25) gives $\upsilon^*(t)$. To find w^*, substitute $\upsilon/D(t)$ from (13) into (19) and solve for w. This gives: $w^* = A_t(1 - \phi_t)\iota^*/(1+\mu\iota^*)$.

3. COMPARATIVE STATICS

Using the LM and RD^{CE} conditions and Figure 1, it is straightforward to establish the following:

Proposition 1: *The steady-state innovation rate* ι^*

- *increases with the innovation size* λ*, the R&D subsidy rate* ϕ_t*, the population growth rate* n*,*
- *decreases with the population share of specialized labor* s*, the subjective discount rate* ρ*, and the resource requirement in R&D* $A_t \equiv a_t\delta/\gamma$ *,*
- *changes in an ambiguous direction with the rate at which DTOs accumulate* μ*.*

Proposition 1 implies that the steady-state growth rate is fully endogenous. The innovation rate ι^* is a function of all of the model's parameters including the R&D subsidy rate. Qualitatively, these results mirror those of Dinopoulos and Syropoulos (2007); however, Dinopoulos and Syropoulos (2007) do not have results for μ. On the other hand, these results differ in a major way from Segerstrom (1998) where $\iota^* = n/\mu$ and thus variations in γ, s, and δ exert no influence on ι^*.[15]

To highlight the new features of my model, I discuss only the shock to μ. An increase in μ exerts two competing effects on R&D profitability. First, it increases the marginal cost of R&D by raising the level of R&D difficulty $D(t)$. Second, it increases the rewards from R&D by reducing the innovation deterring elasticity $\eta(\iota)$ and thereby the effective replacement rate. The net impact on innovation profitability and thus on the RD^{CE} curve is ambiguous. On the other hand, in the general-purpose labor market a larger μ raises $D(t)$, and for a given c, this leaves fewer resources for innovation. Thus, the LM curve shifts to the left. The ambiguous effect of an increase in μ differs from Segerstrom (1998) where $d\iota^*/d\mu < 0$. In my model, a larger μ introduces a new effect by reducing the innovation-deterring elasticity and thereby raising the value of a successful firm. If this effect is

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[14] Note that on the LM curve, as $\iota \to 0$, $c \to \lambda(1 - s)$ and as $\iota \to \iota^{max} = n/\mu$, $c \to -\infty$. On the RD^{CE} curve, as $\iota \to 0$, $c \to c^0 = \lambda s A_t(1 - \phi_t)(\rho - n)/(\lambda - 1)n$ and as $\iota \to \iota^{max} = n/\mu$, $c \to \infty$. Hence, for a unique equilibrium, we need to have the intercept of the LM curve be strictly higher than that of the RD^{CE} curve: $\lambda(1 - s) > c_0 \Rightarrow \lambda - 1 > [sA_t(1 - \phi_t)(\rho - n)]/[(1 - s)n]$

[15] Even though Segerstrom (1998) predicts that steady-state growth responds only to n and μ, it is worth pointing out that in his model's transition path, all of the parameters play an active role in affecting the endogenous variables. In particular, during the transition phase $d\iota/d\lambda > 0$, $d\iota/d\phi_t > 0$, $d\iota/da_t < 0$, and $d\iota/d\rho < 0$, which are in line with Proposition 1.

sufficiently large then the FE curve shifts right and it becomes theoretically possible to have $d\iota^*/d\mu > 0$. Numerical simulations imply that for a wide range of parameters, the elasticity effect turns out to be quite modest and thus $d\iota^*/d\mu < 0$ holds.

4. WELFARE ANALYSIS

I now consider the problem of a social planner who allocates the economy's resources to maximize consumer's welfare over an infinite horizon as measured by (1). Recall from consumer's static optimization that in each industry consumers buy only the highest-quality good and per capita demand for each good is given by $x(j,\omega,t) = c(t)/\lambda$. Substituting this into (2) gives:

$$log\ u(t) = \int_0^1 log\lambda^{j(\omega,t)} d\omega + log[c(t)/\lambda]. \tag{28}$$

Consider now the social planner's allocation decision of a *given* amount of aggregate R&D resources across industries at time t. The planner's goal is to maximize the growth rate of the first term in (28) $\frac{d\int_0^1 log\,\lambda^{j(\omega,t)}d\omega}{dt} = log\,\lambda\int_0^1 \iota(\omega,t)d\omega = log\,\lambda\int_0^1 \frac{R(\omega,t)}{D(\omega,t)}d\omega$, where I have used (6) for $\iota(\omega,t)$. Hence, for a given level of $X(\omega,t)$, the planner devotes all R&D resources to industries with the lowest $D(\omega,t)$; similarly, for a given level of $\iota(\omega,t)$, the planner devotes all specialized labor to industries with the lowest $D(\omega,t)$. Over time, this will imply $D(\omega,t) = D(t)$, $\iota(\omega,t) = \iota(t)$, $X(\omega,t) = X(t)$ for all ω and t. With $\frac{d\int_0^1 log\,\lambda^{j(\omega,t)}d\omega}{dt} = \iota(t)log\,\lambda$, it follows that $\int_0^1 log\lambda^{j(\omega,t)}d\omega = log\lambda\Phi(t)$, where $\Phi(t) = \int_0^t \iota(\tau)d\tau$ stands for the expected number of innovations before time t. Thus, the instantaneous utility at time t captured by (28) boils down to:

$$log\ u(t) = \Phi(t)log\lambda + log[c(t)/\lambda]. \tag{29}$$

Using the general-purpose labor market condition along with $D(\omega,t) = D(t)$ and $\iota(\omega,t) = \iota(t)$ and measure one of industries, it follows that $c(t)/\lambda = (1-s) - a_\iota d(t)\iota(t)$, where $d(t) \equiv D(t)/N(t)$ stands for per capita R&D difficulty. Substituting (29) into (1) using the expression for $c(t)/\lambda$, I can now state the social planner's problem as:

$$\max_{\{\iota\}} \int_0^\infty e^{-(\rho-n)t}\{\Phi(t)log\,\lambda + log[1-s-a_\iota d(t)\iota(t)]\}dt \tag{30}$$

subject to the state equations $\dot{\Phi} = \iota(t)$ and $\dot{d} = [\delta s/\gamma] + \mu\iota(t)d(t) - nd(t)$; the initial conditions $\Phi(0) = 0$, $d(0) = d_0 > 0$; and the control constraint, $(1-s)/a_\iota d(t) \geq \iota(t) \geq 0$ for all t. To derive the $\dot{d}$ equation I have used (7), (21), $\dot{N}/N = n$, along with $\iota(\omega,t) = \iota(t)$, $X(\omega,t) = X(t)$ and measure one of industries.

I solve this optimization problem in Appendix B. I find that there exists a unique balanced growth solution for c and ι characterized by (27) and the *socially optimum R&D condition*:

$$\frac{\delta s}{\gamma(n-\iota\mu)}\frac{a_\iota \rho}{(\rho-\iota\mu)}=\frac{[\log\lambda/\lambda]c}{\rho-n}. \qquad \mathbf{RD^{SO}} \qquad (31)$$

Observe that the $\mathrm{RD^{SO}}$ equation is the analog of $\mathrm{RD^{FE}}$, this time though, the marginal cost of and marginal returns from R&D are measured from the perspective of the social planner. In particular, the $\mathrm{RD^{SO}}$ and $\mathrm{RD^{FE}}$ conditions differ with respect to three terms:

- $\log\lambda$ as the consumer's valuation of a higher quality good in $\mathrm{RD^{SO}}$ versus $\lambda-1$ as the profit margin enjoyed by a successful innovator in $\mathrm{RD^{CE}}$,
- $\rho-n$ as the discount factor of a representative household in $\mathrm{RD^{SO}}$ versus $\rho-n+\iota[1+\eta(\iota)]$ as the replacement-rate-adjusted discount factor of a quality leader in $\mathrm{RD^{CE}}$,
- $\rho/[\rho-\iota\mu]>1$ as a coefficient that magnifies the social planner's perceived R&D cost in $\mathrm{RD^{SO}}$ versus a coefficient of unity in $\mathrm{RD^{CE}}$.

With $\lambda>1$ and $\iota>0$, it is clear that the socially optimum and competitive equilibrium outcomes may differ. Let ~ represent the socially optimal levels of endogenous variables. $\mathrm{RD^{SO}}$ and LM conditions determine $\tilde{\iota}$ and $\tilde{c}$, and $\mathrm{RD^{CE}}$ and LM conditions determine the competitive equilibrium levels ι^* and c^*. To replicate the socially optimum outcome, the optimal R&D policy must imply $\iota^*=\tilde{\iota}$ and $c^*=\tilde{c}$. Given that the LM condition is the same, this simply requires that $\mathrm{RD^{CE}}$ imply $\iota^*=\tilde{\iota}$, in which case $c^*=\tilde{c}$ would hold automatically. Substituting for c from $\mathrm{RD^{SO}}$ into $\mathrm{RD^{CE}}$ and using the expression for $\eta(\iota)$ gives the equation that characterizes the optimal subsidy rate ϕ_ι^{SO} as:

$$(1-\phi_\iota^{SO})\frac{\log\lambda}{\rho-n}=\frac{(\lambda-1)}{\left[\rho-n+\iota\left(1+\frac{n-\iota\mu}{1+\mu\iota}\right)\right]}\frac{\rho}{(\rho-\iota\mu)} \qquad (32)$$

It is easy to see from (32) that depending on the parameters of the model, the optimal policy can be a tax $\phi_\iota^{SO}<0$, or a subsidy $0<\phi_\iota^{SO}<1$. For a generic parameter α, it follows that $d\phi_\iota^{SO}/d\alpha = \partial\phi_\iota^{SO}/\partial\alpha + (\partial\phi_\iota^{SO}/\partial\iota)(d\iota/d\alpha)$, where the partials come from (32) and $d\iota/d\alpha$ comes from Proposition 1. One can show that $\partial\phi_\iota^{SO}/\partial\iota >< 0$ and further substitution of the partial derivatives do not resolve the ambiguity.

4.1. Marginal Welfare Analysis

To understand the forces at work, I follow Grossman and Helpman (1991) and Segerstrom (1998) and consider the effects of a marginal innovation by an external entrepreneur on welfare as measured by (1). This equals (see Appendix C for the derivation):

$$MU_{\Phi} = \frac{dU}{d\Phi} = \underbrace{\frac{\log\lambda}{\rho - n}}_{\text{CS}} - \underbrace{\frac{\lambda - 1}{\rho - n + \iota\left[1 + \eta(\iota)\right]}}_{\text{BS}} - \underbrace{\overbrace{\frac{\lambda - 1}{\rho - n + \iota\left[1 + \eta(\iota)\right]}}^{\text{BS}} \times \overbrace{\frac{\iota\mu}{(\rho - \iota\mu)}}^{\text{Spillover}}}_{\text{IS}} \tag{33}$$

The first term in (33) measures the *consumer surplus externality* (henceforth **CS**). With each additional innovation, consumers enjoy a higher level of utility because product quality increases and yet prices remain constant. Furthermore, these utility gains accumulate over time because each successful innovation adds to the knowledge base and paves the way for the subsequent R&D race that is aimed at innovating the next-generation product. Entrepreneurs do not take into account in their R&D decisions these utility gains that accrue to consumers over an infinite horizon. Hence, the CS effect captures *a positive externality* associated with an additional innovation, calling for an R&D subsidy.

The second term in (33) measures the *business stealing externality* (henceforth **BS**). For each industry, successful innovation implies the replacement of the incumbent producer with a new quality leader. As a result, the stockholders of the incumbent firm suffer a loss in their asset valuations, which equals the expected discounted value of the forfeited stream of monopoly profits. Consequently, incomes and consumer expenditures decline for all industries. This creates a multiplier effect, further lowering incomes and expenditures and so on. Entrepreneurs do not take into account in their R&D decisions the losses incurred by the incumbent firms and its reverberations throughout the economy. Thus, the BS term measures *a negative externality* associated with an additional innovation, calling for an R&D tax.

The third term in (33) measures the *intertemporal R&D spillover externality* (henceforth **IS**) associated with DTOs. R&D investment by entrepreneurs in the current period leaves ex-ante less promising projects for future entrepreneurs and raises the difficulty of research in the subsequent periods. This implies that more resources are devoted to R&D and fewer resources remain for the production of final goods. In equilibrium, lower production translates into lower consumption expenditure and thus lower profits. This triggers a multiplier effect, further decreasing incomes and expenditures and so on. Entrepreneurs do not take into account in their R&D decisions the negative implications of their current research activities for future innovation efforts and their reverberations throughout the economy. Thus, the IS term captures a *negative externality* associated with marginal innovation, calling for R&D taxes. The first component of the IS externality is essentially the BS externality. The second component is the *spillover component*, which is increasing with $\iota\mu$, the rate at which DTOs accumulate within each industry.

In short, with DTOs and RPAs modeled simultaneously, the current paper identifies three welfare externalities. The effective replacement rate term $\iota[1+\eta(\iota)]$ turns out to be a part of the BS and IS externalities, with $\eta(\iota)$ capturing the interactions between RPAs and DTOs. Since growth is fully-endogenous, all of the model's parameters affect the magnitudes of the externalities through their influence on ι and also on $\eta(\iota)$.

How does the MU_{Φ} term above compare with the literature? In Dinopoulos and Syropoulos (2007), there are no DTOs ($\mu = 0$); thus, the IS term is entirely absent. In addition, the innovation deterring elasticity is $\eta(\iota) = 1$. Hence, RPAs exerts a rigid impact on MU_{Φ}. Similar to the current model though, in Dinopoulos and Syropoulos ι responds to all of the model's parameters; hence parametric shocks influence MU_{Φ} through this channel. In Segerstrom (1998), all three welfare externalities are present; however, there are no RPAs ($\delta = 0$), which implies $\eta(\iota) = 0$. This means that the effective replacement rate equals ι. Moreover, in Segerstrom, $\iota = n/\mu$; thus, parametric shocks exert no influence on MU_{Φ} through the ι channel (of course with the exception of shocks to n and μ). In both Dinopoulos and Syropoulos (2007) and Segerstrom (1998) the interaction between DTOs and RPAs are absent by construction.

5. SIMULATION RESULTS

The analytical exposition implies that the optimal R&D policy ϕ_{ι}^{SO} can be a tax or a subsidy depending on the parameters of the model. Thus, I run numerical simulation to determine the optimal level of ϕ_{ι}^{SO}. I choose the following benchmark parameters:

$\lambda = 1.25$, $\rho = 0.07$, $n = 0.01$, $s = 0.00023$, $A_{\iota} = 70$, $\mu = 0.20$.

The size of innovations, λ, measures the gross mark up (the ratio of the price to the marginal cost) enjoyed by innovators and is estimated as ranging between 1.05 and 1.4 [see Basu, 1996, and Norrbin, 1993]. The population growth rate, n, is calculated as the annual rate of population growth in the US between 1975 and 1995 according to the World Development Indicators (World Bank, 2003). The subjective discount rate, ρ, is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985). The percentage of specialized labor s is set at 0.023 percent to generate a fifty percent wage differential between specialized and general-purpose labor. The goal is to capture the relatively higher earnings of lobbyists/lawyers with respect to other workers. The resource requirement parameter for innovation, A_{ι}, is set at 70 to generate a growth rate $g = \iota log\lambda$ in the neighborhood of 0.5 percent. This is due to Denison (1985) who calculates the rate of growth driven by knowledge advancements to be around 0.5 percent. The choice of $\mu = 0.20$ follows from Dinopoulos and Segerstrom (1999) and Steger (2003).[16]

The benchmark simulations taken at face value imply that the competitive equilibrium innovation rate is ι^{CE} =0.0218425 and the employment share of R&D workers with respect to general-purpose labor is s_R^{CE} = 0.0624459. These values are below the socially optimal levels ι^{SO} = 0.0247148 and s_R^{SO} = 0.078684. Obviously, the optimal R&D policy is a subsidy and turns out to equal ϕ_ι^{SO} = 0.147461. How does ϕ_ι^{SO} change with λ and other parameters? To answer this question, I map ϕ_ι^{SO} against λ and considered high and low values for *each* parameter within a 30 percent range as a rule of thumb. The simulations reveal a robust n-shaped relationship between ϕ_ι^{SO} and λ as shown in Figure 2, meaning that for very small and very large values of λ, it is optimal to tax R&D; and for medium-size values of λ it optimal to subsidize R&D. 3D numerical simulations which are presented in Appendix D further confirm the robustness of this n-shaped relationship. [17]

Figure 2 implies that the direction and magnitude of optimal R&D policy is highly sensitive to the choice of parameters. The question then is: would it be possible to identify a plausible range to pin down the level of optimal R&D policy? For this purpose, I choose a parameter range that keeps the calibrated values of the model's central endogenous variables within an empirically relevant band. I consider these variables to be: *i)* the steady-state growth rate g, and *ii)* the steady-state employment share of R&D workers s_R.[18] After all, the model at hand is an endogenous growth model that links economic growth to the amount of resources allocated to R&D.

I propose in particular the following 3-step methodology. First, run a benchmark simulation using average values/estimates from the literature and adjust the free parameter A_ι to set $g = 0.005$ (as already done in above). Second, identify a range for λ that keeps s_R below 0.07. As illustrated in Figure 2, λ plays a pivotal role in determining the direction of R&D policy; hence, the analysis should allow for a certain amount of variation in λ. To this end, I choose the interval $\lambda \in [1.20, 1.35]$ which keeps s_R within the interval [0.03, 0.07].[19] Third, given the range for λ, perturb each parameter such

[16] In general, the benchmark parameters and outcomes are in line with the recent theoretical growth papers that use numerical simulations [see Jones, 2002, Lundborg and Segerstrom, 1999, Sayek and Şener, 2006, Şener, 2006, Dinopoulos and Segerstrom, 1999, Steger, 2003, and Segerstrom 2007].

[17] In the working paper Şener (2007), I provide a detailed analytical discussion of how welfare externalities respond to changes in parameters.

[18] Segerstrom (2007) strongly emphasizes the importance of generating empirically relevant levels for s_R.

[19] This range for s_R is above the observed level for the US and other advanced countries, which is in the neighborhood of 0.01. However, it is well-known that the R&D employment intensity definition is too narrow to account for all employment involved in creating, refining and disseminating new ideas. In Jones' [(2002), p. 226] words, "the research behind the creation of new consumer products like Odwalla or Jamba juice fruit drinks

that the rate of innovation ι remains within the interval [0.01, 0.04]. This allows for some of the variation to emanate from the parameter of interest. [20] The simulation results for the restricted range $\lambda \in [1.20, 1.35]$ are shown in Figure 3. In most cases the optimal R&D policy is a subsidy and lies within a 5-25 percent band.[21] This finding is in line with the existing literature which tends to recommend subsidizing R&D while recognizing the competing welfare effects of a marginal innovation.[22]

It is also worthwhile to investigate the responsiveness of the innovation rate to changes in the subsidy rate. For the benchmark case, I found that a 10 percentage points increase in ϕ_ι (from 0 to 0.10) leads to an 8.4 percent increase in ι^*. Table 1 presents the outcome of this exercise for high and low parameter values, consistent with the range in Figure 3. The results show that the responsiveness of ι ranges between 2.9 percent and 13.1 percent. Hence, R&D policies can have a sizeable quantitative impact on the rate of innovation.

6. A VARIANT OF THE BASELINE MODEL

In the long-run, workers make decisions about acquiring skills and respond to incentives. This leads to labor mobility across activities and endogenizes the share of labor allocated to RPAs. To capture this mechanism, I consider a setting where labor is of one type and fully mobile across manufacturing, R&D and RPAs. I will thus refer to this setting as the "endogenous *s*" case and refer to the previous setting as the "baseline" model. [23] The technical details are in Appendix E.

Normalizing the wage rate of labor to one and combining (13) and (19) to obtain the equilibrium innovation rate gives:

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is not included" in this definition. Moreover, in the US, the employment intensity measure focuses on science and engineering, emphasizing research that requires the equivalent of a 4-year degree. In Jones' words "the research undertaken by the young Steve Jobs, Bill Gates and Marc Andreessen was probably excluded" from this statistics.

[20] This also allows for a considerably wide range for the growth rate $g \in [0.0018232, 0.0120042]$. To see this, note that $g = \iota \log\lambda$; thus $g^{LOW} = 0.01*\log[1.2] = 0.0018232$ and $g^{HIGH} = 0.04*\log[1.35] = 0.0120042$. Recall that the estimate for long-run growth attributable to technology advancements is 0.5% [Denison, 1995], and the average US per-capita income growth rate over the last 125 year is a steady 1.8% [Jones, 2002].

[21] The only exceptions are for the upper bound values of ρ and μ. When $\rho = 0.0095$, the subsidy rate is positive but clearly below 0.05. When $\mu = 0.4$, the subsidy rate is only slightly above zero for $\lambda < 1.3$ and becomes negative (hence a tax) for $\lambda < 1.3$.

[22] See among others Segerstrom (2007), Alvarez-Pelaeza and Groth (2005), Li (2001), Jones and Williams (2000), Stokey (1995), Romer (1990), and the variety-expansion based model of Grossman and Helpman (1991). See Jones and Williams (1998) for an empirical paper that reports that actual R&D investment in the US is 25 to 50 percent of the optimal R&D investment. It should be noted that Dinopoulos and Syropoulos (2007) do not provide a quantitative evaluation of their model; hence, the present paper is the first attempt to quantify optimal R&D policy in an endogenous growth setting with RPAs.

[23] I am thankful to a referee for suggesting the "endogenous *s*" setting.

$$\iota^* = \frac{1}{A_t(1-\phi_t)-\mu}.$$

This implies that growth is endogenous in the sense that the R&D subsidy rate ϕ_t can affect ι^*. However, ι^* is not a function of all of the model's parameters; hence, in that sense growth is *not* fully endogenous. In particular ι^* is now unaffected by changes in ρ, n and λ. The reason is that with only one type of labor, the relative profitability of R&D relative to RPA is no longer tied to a relative wage [to see this take the ratio of (13) to (19)]. The absence of such a link eliminates the feedback effects of ρ, n and λ on ι^*. Adding one more type of labor in either R&D/RPA or both reestablishes the link between relative profitability and relative wage, tying ι^* to all of the model's parameters again.

To investigate welfare, I consider a social planner who maximizes welfare by choosing ι, taking x as given. The planner has at her disposal only R&D subsidies/taxes but no policy tools for RPAs. [24] The level of x is determined by the optimization decisions of the monopolists. To determine the value of ϕ_t^{SO}, I run numerical simulations. I keep the benchmark parameters of Section 5 with two exceptions. The parameter s is omitted because labor is of one type, and A_t is set at 45 to ensure that $g = 0.005$. The simulations imply that the competitive equilibrium innovation rate is $\iota^{CE} = 0.0223214$. Let s_M, s_{RD}, and s_X represent the employment share of manufacturing, R&D, and RPA workers in total labor force, respectively. The competitive equilibrium levels are $s_M{}^{CE} = 0.9362776$, $s_R{}^{CE} = 0.0633731$ and $s_X{}^{CE} = 0.0003492$. The socially-optimal levels are $\iota^{SO} = 0.0266409$, $s_M{}^{SO} = 0.91584$, $s_R{}^{SO} = 0.0838336$, $s_X{}^{SO} = 0.0003266$. Clearly, $\iota^{SO} > \iota^{CE}$; hence, the optimal policy is a subsidy which is calculated as $\phi_t^{SO} = 0.161415$.

The benchmark simulations of the endogenous s case imply that the competitive markets underinvest in R&D and overinvest in manufacturing and RPAs. Replicating the socially optimal outcome requires that resources be drawn from *both* manufacturing and RPA to increase the employment share of R&D workers. When ϕ_t^{SO} is mapped against λ, a downward sloping curve emerges as shown in Figure 4. This is of course different from the n-shaped curve found in the baseline model. The reason is that with ι^{CE} independent of λ, it immediately follows from that (33) that $\mathrm{d}MU_\Phi/\mathrm{d}\lambda < 0$. Not surprisingly, this is similar to Segerstrom (1998), where $\iota^{CE} = n/\mu$, and ϕ_t^{SO} is unambiguously decreasing in λ. Figure 4 shows that when we restrict attention to a reasonable range

[24] If one instead assumes that tax/subsidy policies are available for both rent protection and R&D activities, then the model does not generate any new insights. The reason is that in this case the social planner taxes RPAs prohibitively high, ensuring that no labor is allocated for RPAs. The model then effectively boils down to that of Segerstrom (1998) as far as welfare implications are concerned. It should be noted that in the real world it is hard to identify RPAs and implement targeted policies. Thanks are to a referee for suggesting to investigate the case with no policy tools for RPAs.

for λ and the parameter in question, the optimal policy is a subsidy and lies roughly between 5-25 percent, as in the baseline model. As for the quantitative impact of R&D policies, I find that a 10 percentage point increase in ϕ_ι (from 0 to 0.1) leads to a 10.5 percent increase in ι. This turns out to be a stable value changing by no more than a 0.5 percentage points when parameters are allowed to vary within the range used for Figure 4.[25]

6.3. A Discussion on Welfare Externalities

It should be acknowledged that the model does not account for all of the conceivable externalities associated with a marginal innovation. In Şener (2007), I considered the effects of "*inter-industry knowledge spillovers*" (à la Li, 2003) and "*inter-industry rent-protection spillovers*" (a new channel) on welfare externalities. I find that the spillover parameters can affect the magnitudes of BS and IS externalities directly and also indirectly via their impact on ι. However, the main results of the paper remained intact. Li (2003) constructs a quality-ladders growth model with a CES utility function and removes the scale effects by assuming that R&D difficulty increases as the products become more complex with each innovation and also allows for DTO effects. Li's model introduces two additional welfare externalities. One is the "*across-industry business stealing externality*" by which new innovations reduce the profit flow of leaders in other industries through lowering consumer demand. The other is an effect that reinforces "*the intertemporal R&D spillover externality*". Each innovation success adds to the product complexity and raises R&D difficulty in the subsequent periods. In addition, with CES preferences Li's model allows for unconstrained monopoly pricing for large-sized innovations (instead of limit pricing). This links product prices to the elasticity of substitution, limiting the role of innovation size in affecting the BS externality through profit margins.

Segerstrom (2007) uses Li's (2003) setting to construct a model in which both incumbent firms and outside entrepreneurs undertake R&D. Segerstrom's (2007) model allows for high quality products to be copied at an exogenous rate. He finds that as the rate of copying and hence the rate of replacement increases, the magnitudes of the negative externalities (the intertemporal R&D spillover effect, and the across and within business stealing effects) decline and the optimal R&D policy moves toward a subsidy. Etro (2008) builds a growth model with Stackelberg competition in R&D which also allows for R&D investment by both incumbents and outsiders. He finds that it is always optimal to subsidize R&D. Jones and Williams (2000) construct a model of variety-expansion based endogenous growth model that incorporates DTOs. They introduce creative destruction by assuming a link

[25] I also consider a setting where RPAs use both general-purpose and specialized labor. Not surprisingly the results turn out to be between the "baseline" and the "endogenous *s*" models. As the wage bill share of specialized labor increases, the n-shaped curve resurfaces. See Appendix F for complete technical details.

between old and new varieties via "*innovation clusters*". With this mechanism, the successful innovator of a new variety can claim market share from existing producers—in addition to her own monopoly profits—by exploiting the technology link between the new variety and existing products. Jones and Williams also allow for a negative externality associated with R&D duplication, labeled as the "*stepping on toes effect.*" Incorporating the above mentioned mechanisms into the present paper's setting can be multiple directions for further research.[26]

7. CONCLUSION

This paper has constructed a scale-free quality-ladders endogenous growth model that combines two of the main approaches to removal of scale effects: the RPA approach (à la Dinopoulos and Syropoulos, 2007) and the DTO approach (à la Segerstrom, 1998). I find that the steady-state equilibrium growth rate is a function of all of the model's parameters including the R&D subsidy rate. Hence, the model implies fully-endogenous growth. The presence of RPAs augments the effective replacement rate faced by the incumbent firms. The magnitude of this augmentation is positively related to innovation-deterring elasticity, which is endogenously determined.

The optimal R&D policy exhibits an n-shaped relationship with respect to innovation size. When innovations are of very small and very large magnitudes, the optimal policy is a R&D tax, and for medium size innovations, the optimal policy is a R&D subsidy. Numerical simulations imply that competitive markets typically underinvest in R&D and thus the optimal R&D policy is a subsidy. The magnitude of the R&D subsidy lies between 5 to 25 percent for plausible parameter values.

How do these finding compare with the real world R&D subsidy rates? For the OECD countries, the average percentage of business enterprise R&D expenditure funded by the government is in the neighborhood of 10% (OECD, 2000, p.31). Hence, the developed countries may not be far off from their optimal levels but yet there may be room for pushing the R&D subsidy rates upward to maximize welfare. The simulations of the present paper cannot resolve this magnitude issue once and for all; however, the findings strengthen the case for R&D subsidies and may pave the way for future research aimed at fine tuning the magnitude of optimal R&D policy.

Several extensions of the model still remain to be explored. One can incorporate the variety expansion approach to this setting and study the implications for R&D policy. One can introduce human capital and physical capital accumulation and check the robustness of the main results [see for instance Strulik (2007) and Steger (2005)]. Finally, one can extend the model to a two-country setting and investigate the effects of intellectual property and tariff policies on economic growth.

[26] See also Li (2001) and O'Donoghue and Zweimuller (2004) for models that consider patent breadth in the context of optimal R&D policy.

REFERENCES

AIPLA, 1997. Report of Economic Survey. American Intellectual Property Law Association. Washington, DC.

Aghion, P., Howitt, P., 1992. A model of growth through creative destruction. Econometrica 60, 323-351.

Aghion, P., Howitt, P., 1998, Endogenous Growth Theory. MIT Press, Cambridge, MA.

Alvarez-Pelaeza, M. J., Groth, C., 2005. Too little or too much R&D? European Economic Review 49, 437-456.

Basu, S., 1996. Procyclical productivity: Increasing returns or cyclical utilization. Quarterly Journal of Economics 111, 709-51.

Cohen, W., Nelson, R., Walsh, H., 2000. Protecting their intellectual assets: Appropriability conditions and why US manufacturing firms patent (or not). National Bureau of Economic Research Working Paper 7552.

Denison, E., 1985. Trends in American Economic Growth, 1929-1982. The Brookings Institution, Washington, DC.

Dinopoulos, E., Segerstrom P., 1999. A Schumpeterian model of protection and real wages. American Economic Review 89, 450-72.

Dinopoulos, E., Thompson, P., 1998. Schumpeterian growth without scale effects. Journal of Economic Growth 3, 313-335.

Dinopoulos, E., Thompson, P., 1999. Scale effects in Schumpeterian models of economic growth. Journal of Evolutionary Economics 9, 157-185.

Dinopoulos, E., Syropoulos, C., 2007. Rent protection as a barrier to innovation and growth. Economic Theory 32, 309-332.

Dinopoulos, E., Şener, F., 2007. New directions in Schumpeterian growth theory. In: Horst Hanusch and Andreas Pyka (Eds.), Elgar Companion to Neo-Schumpeterian Economics. Edward Elgar, Cheltenham, UK, pp. 688-704.

Etro, F., 2008. Growth leaders. Journal of Macroeconomics. Forthcoming.

Grossman, G., Helpman, E., 1991, Innovation and Growth in the Global Economy. MIT Press, Cambridge, MA.

Ha, J., Howitt, P., 2007. Accounting for trends in productivity and R&D: A Schumpeterian critique of semi-endogenous growth theory. Journal of Money, Credit, and Banking 39, 733-774.

Howitt, P., 1999. Steady endogenous growth with population and R&D inputs growing. Journal of Political Economy 107, 715-730.

Jones, C., 1995a. Time series tests of endogenous growth models. Quarterly Journal of Economics 110, 495-525.

Jones, C., 1995b. R&D-based models of economic growth. Journal of Political Economy 103, 759-784.

Jones, C., Williams, J. 1998. Measuring the social return to R&D. Quarterly Journal of Economics 113, 1119-1135.

Jones, C., 1999. Growth: With or without scale effects? American Economic Review Papers and Proceedings 89, 139-144.

Jones, C., Williams, J., 2000. Too much of a good thing? The economics of investment in R&D. Journal of Economic Growth 5, 65-85.

Jones, C., 2002. Sources of US economic growth in a world of ideas. American Economic Review 92, 220-239.

Jones, C., 2005. Growth and ideas. In: Aghion, P., Durlauf, S.N. (Eds.), Handbook of Economic Growth. Amsterdam, North-Holland, pp. 1064-1111.

Kortum, S., 1997. Research, patenting and technological change. Econometrica 65, 1389-1419.

Lerner, J., 1995. Patenting in the shadow of competitors. Journal of Law and Economics 38, 463-496.

Levin, R., Klevorick, A., Nelson, R. and Winter, S., 1987. Appropriating the Returns of Industrial R&D. Brookings Papers on Economic Activity, 783-820.
Li, C., 2001. On the policy implications of endogenous technological progress. Economic Journal 111, C164-179.
Li, C., 2003. Endogenous growth without scale effects: A Comment. American Economic Review 93, 1009-1017.
Lundborg, P., Segerstrom, P., 2002. The growth and welfare effects of international mass migration. Journal of International Economics 56, 177-204.
Mehra, R., Prescott, E.C., 1985. The equity premium: A puzzle. Journal of Monetary Economics 15, 145-61.
Norrbin, S. C., 1993. The relationship between price and marginal cost in US industry: A contradiction. Journal of Political Economy 101, 1149-64.
O'Donoghue, T., Zweimuller, J., 2004. Patents in a model of endogenous growth. Journal of Economic Growth 9, 81-123.
OECD, 2000. Science, Technology and Industry Outlook, Paris.
Peretto, P., 1998. Technological change and population growth, Journal of Economic Growth 3, 283-312.
Romer, P., 1990. Endogenous technological change. Journal of Political Economy 98, S71-102.
Sayek, S., Şener, F., 2006. Outsourcing and wage inequality in a dynamic product cycle model. Review of Development Economics 10, 1-19.
Şener, F., 2006. Intellectual property rights and rent protection in a North-South product cycle model. Mimeo, Union College.
Şener, F., 2006. Labor Market Rigidities and R&D Based Growth in the Global Economy. Journal of Economic Dynamics and Control 30, 769-805.
Şener, F., 2007. R&D policies, endogenous growth and scale effects. Working Paper, Union College.
Segerstrom, P., 2007. Intel economics. International Economic Review 48, 247-280.
Segerstrom, P., 1998. Endogenous growth without scale effects. American Economic Review 88, 1290-1310.
Segerstrom, P., Anant, TCA., Dinopoulos, E., 1990. A Schumpeterian model of the product life cycle. American Economic Review 80, 1077-1092.
Somaya, D., 2002, Patent Litigation in the United States (1970-200). University of Maryland, mimeo.
Steger, T. M., 2003. The Segerstrom model: Stability, speed of convergence and policy implications. Economics Bulletin 15, 1-8.
Steger, T. M., 2005. Welfare implications of nonscale R&D-based growth models. Scandinavian Journal of Economics 107, 737-757.
Stokey, N., 1995. R&D and economic growth. Review of Economic Studies 62, 469-489.
Strulik, H., 2007. Too much of a good thing? The quantitative economics of R&D-driven Growth revisited, Scandinavian Journal of Economics 109, 369-386.
Young, A., 1998. Growth without scale effects. Journal of Political Economy 106, 41-63.
World Bank, 2003. World Development Indicators, Washington, DC.
Zachariadis, M., 2003. R&D, Innovation and technological progress: A test of the Schumpeterian framework without scale effects. Canadian Journal of Economics 36, 566-86.
Zachariadis, M., 2004. R&D-induced growth in the OECD?. Review of Development Economics 8, 423-39.

Table 1. Percent change in ι due to a 10 percentage points increase in ϕ_ι

Benchmark	0.08445
$A_\iota = 28$	0.02933
$A_\iota = 92$	0.12640
$\lambda = 1.20$	0.11698
$\lambda = 1.35$	0.05524
$\mu = 0.12$	0.09392
$\mu = 0.40$	0.07466
$n = 0.0082$	0.11344
$n = 0.0132$	0.06105
$\rho = 0.055$	0.06338
$\rho = 0.095$	0.13746
$s = 0.000092$	0.02932
$s = 0.00031$	0.13185

Figure 1. Steady-State Equilibrium

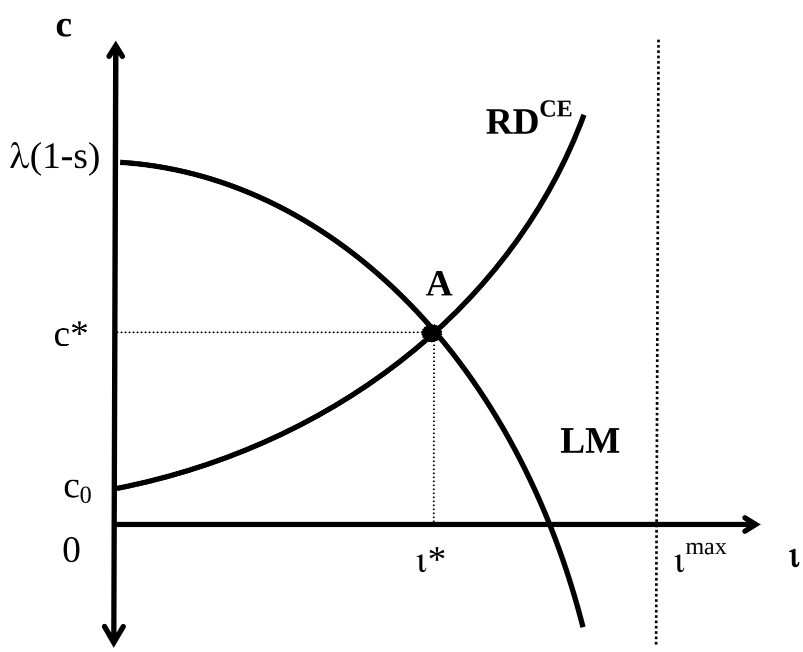
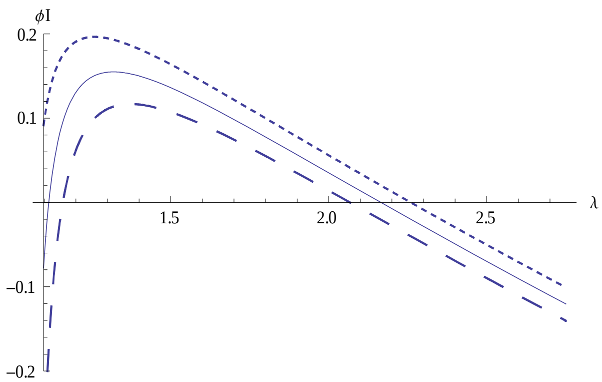
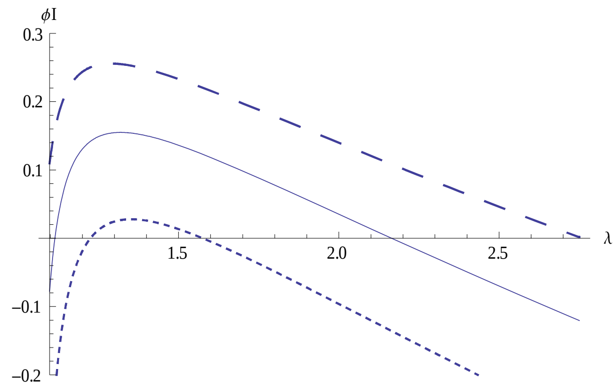


Figure 2: Simulation Results for Optimal Subsidy ϕ_ι^{SO}
(λ unrestricted, range for each parameter +/- 30%)

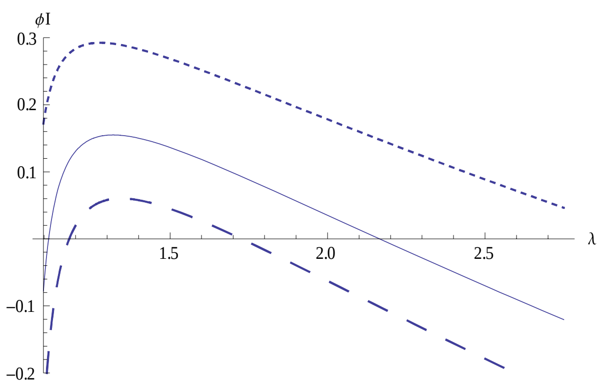
a) $s = 0.00023$ (Δs = +/– 30 %)



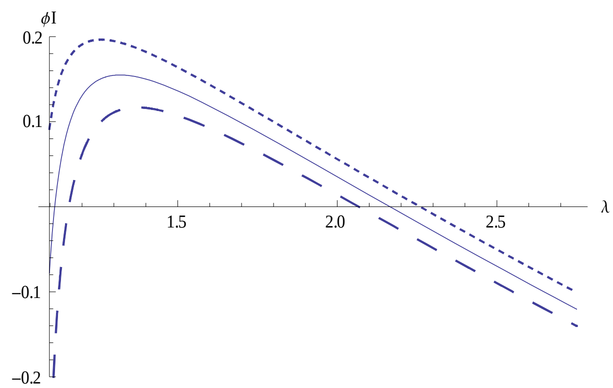
b) $n = 0.01$ (Δn = +/– 30 %)



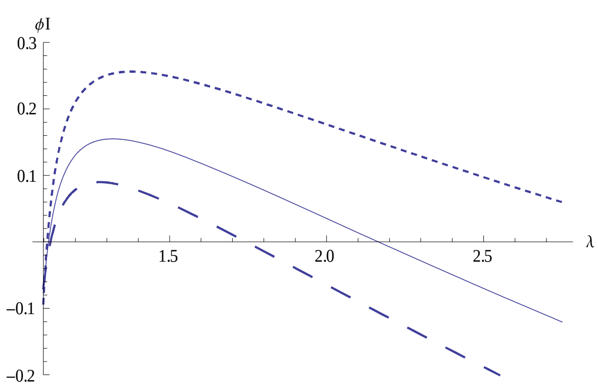
c) $\rho = 0.07$ ($\Delta\rho$ = +/– 30 %)



d) $A_\iota = 70$ (ΔA_ι = +/– 30 %)



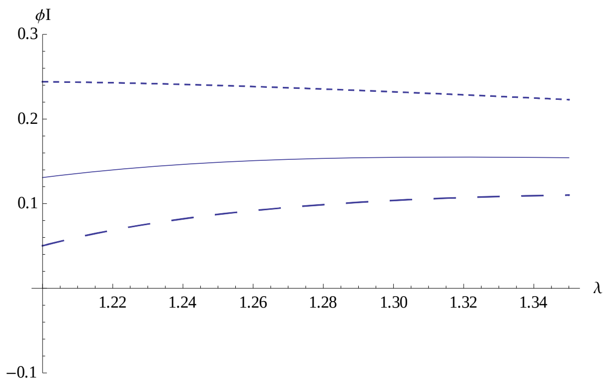
f) $\mu = 0.2$ ($\Delta\mu$ = +/– 30 %)



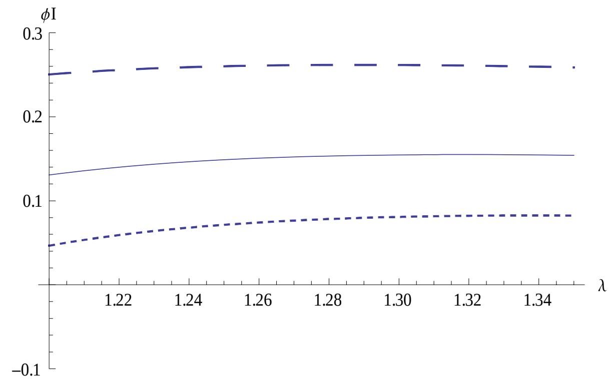
Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.

Figure 3: Simulation Results for Optimal Subsidy ϕ_ι^{SO}
(λ restricted, parameter range contingent on ι impact)

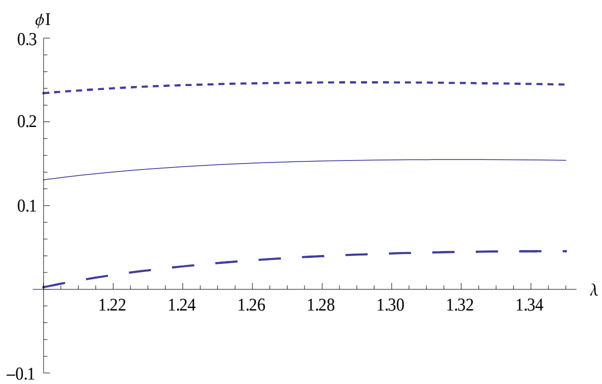
a) $s^L = 0.000092$, $s^H = 0.00031$



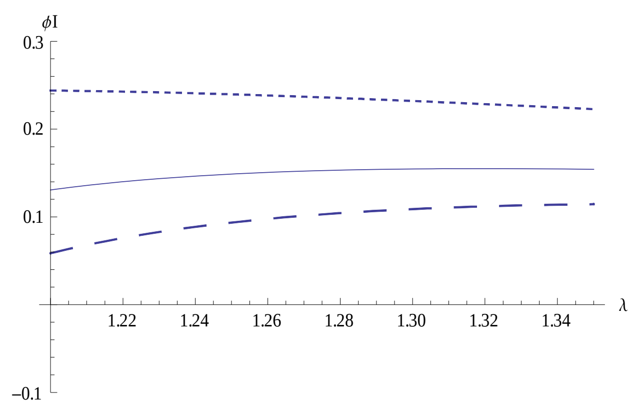
b) e) $n^L = 0.0082$, $n^H = 0.0132$



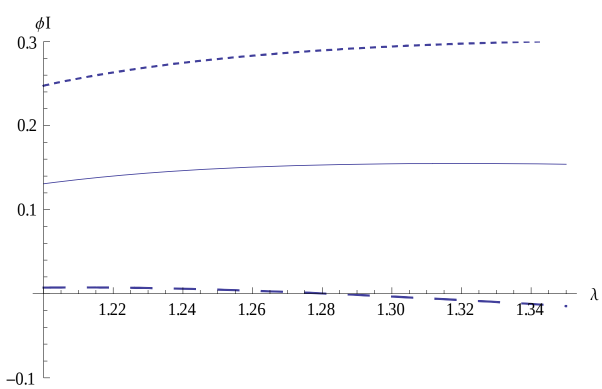
c) $\rho^L = 0.055$, $\rho^H = 0.095$



d) $A_\iota^L = 28$, $A_\iota^H = 92$



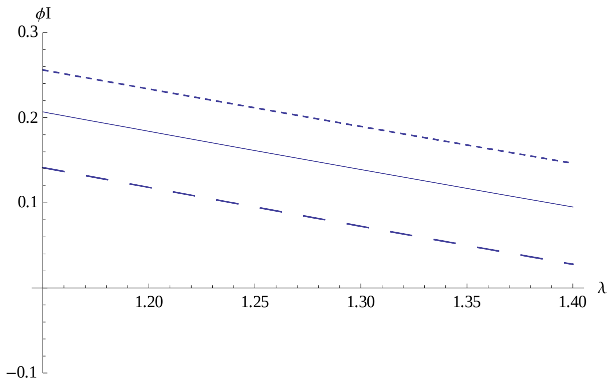
f) $\mu^L = 0.12$, $\mu^H = 0.40$



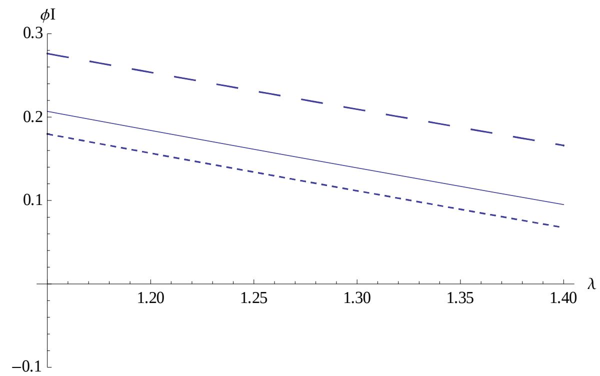
Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.

Figure 4: Simulation Results for Optimal Subsidy ϕ_ι^{SO} with endogenous s
(λ restricted, parameter range contingent on s_R impact)

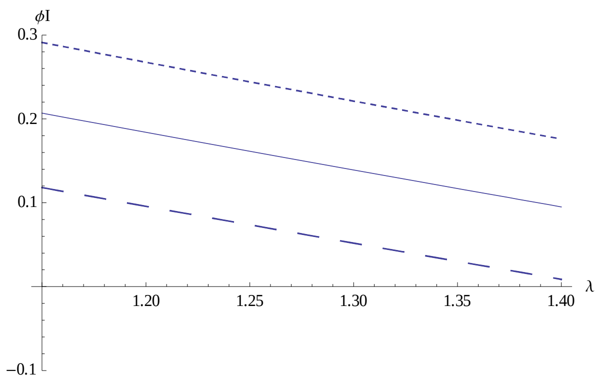
a) $\rho^L = 0.06$, $\rho^H = 0.09$



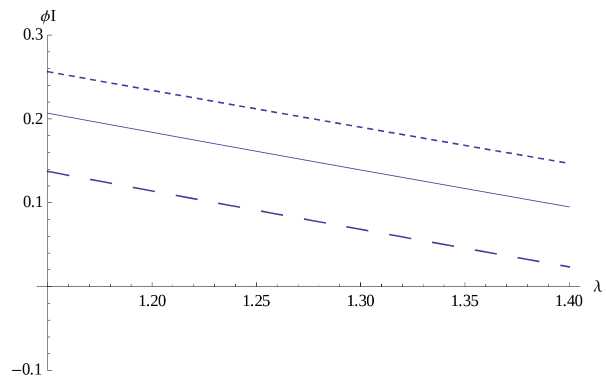
b) $n^L = 0.005$, $n^H = 0.02$



c) $\mu^L = 0.02$, $\mu^H = 0.40$



d) $A_\iota^L = 37$, $A_\iota^H = 62$



Note: Long-dashing for higher parameter value, short-dashing for lower parameter value.