Appendices[*]

for

R&D Policies, Endogenous Growth and Scale Effects

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Appendix A : Proof of Stability for Competitive Equilibrium

I characterize the transitional dynamics of the economy by generating an autonomous system for the equations of motion for c and d, where c is consumption per capita and $d \equiv D/N$ is R&D difficulty per capita. During the transition, the labor market conditions, free-entry in R&D and optimal rent protection activity conditions must hold. I restrict attention to the domain $d(t) > 0$, and $\lambda(1-s) > c(t) > 0$ which ensures $\iota(t) > 0$. Note from equation (13) that $\dot{\upsilon}/\upsilon = \dot{D}/D$. Substituting for $X(\omega, t)$ from (21) into (7) using $\iota = \iota(\omega, t)$ and $X = X(\omega, t)$ and the measure one of industries, it follows that $\dot{D}/D = [(\delta+\delta_A)s / \gamma d] + \iota(\mu - \mu_A)$. Combining (19) with (21), $w\gamma X(t)/N(t)$ can be written as $w\gamma X(t)/N(t) = \delta\iota a_\iota(1-\phi_\iota)s/(1+\mu\iota)\gamma$. Solving equation (11) for $r(t)$ and substituting the resulting expression into (4), using the expressions for $\dot{\upsilon}/\upsilon$ and $w\gamma X(t)/N(t)$ immediately implies:

$$\dot{c} = c\left\{\frac{1}{d}\left[\frac{c(\lambda-1)}{\lambda a_\iota(1-\phi_\iota)} - \frac{\delta s}{\gamma(\mu+(1/\iota(c,d))} + \frac{s(\delta+\delta_A)}{\gamma}\right] - \iota(c,d)\left[1-(\mu-\mu_A)\right] - \rho\right\}.$$

where $\iota(c, d) = [1 - s - (c/\lambda)]/da_\iota$ comes from (20). Observe that $dd/dc|_{\dot{c}=0}$ is indeterminate. On the other hand, it is possible to draw some inferences on the movement of c around $\dot{c} = 0$. More specifically, if $\mu - \mu_A < 1$ then $d\dot{c}/dc > 0$, implying that starting from any point on $\dot{c} = 0$ curve, an increase in c leads to $\dot{c} < 0$. If $\mu - \mu_A > 1$ then $d\dot{c}/dc$ remains ambiguous.

To find the equation of motion for d simply substitute $\iota(c, d)$ into $\dot{d}/d = \dot{D}/D - n$ using the expression for $\dot{D}/D$ from above. This implies:

$$\dot{d} = \frac{(\delta+\delta_A)s}{\gamma} + \frac{(\mu-\mu_A)}{a_\iota}\left[1 - s - \frac{c}{\lambda}\right] - nd .$$

Observe that $dd/dc|_{\dot{d}=0} < 0$ and that $d\dot{d}/dd < 0$. Starting from any point on the $\dot{d} = 0$ line, an increase in d renders $\dot{d} < 0$, and an decrease in d renders $\dot{d} > 0$.

Given the ambiguous relationships coming from $\dot{c}=0$, there are multiple possibilities. However, we know that in the relevant domain, the intersection of $\dot{c}=0$ and $\dot{d}=0$ generates a unique point. Hence, using a graphical approach, it is straightforward to analyze the local stability of this non-linear system. When $\dot{c}=0$ is upward sloping, the system has a stable node if $d\dot{c}/dc > 0$ and a stable focus otherwise. When $\dot{c}=0$ is downward sloping, the system has a stable node independent of the sign of $d\dot{c}/dc$. This result holds regardless of $\dot{c}=0$ being flatter or steeper than $\dot{d}=0$. Therefore, we conclude that the system is locally stable, exhibiting either a stable node or a stable focus. Figure Appendix A plots the demarcation curves $\dot{c}=0$ and $\dot{d}=0$ under the benchmark values for the parameters. Observe that $\dot{c}=0$ is upward sloping and given $\mu-\mu_A<1$ it follows that $d\dot{c}/dc>0$. In this case, the system is saddle path stable. Numerical simulations imply that for a wide range of reasonable parameters the picture depicted in Figure Appendix A remains valid.

Appendix B: Optimal R&D Policy: Solution, Uniqueness and Stability

The social planner's problem is to

$$\max_{\{\iota\}} \int_0^\infty e^{-(\rho-n)t}\{\Phi(t)\log\lambda + \log[1-s-a_\iota d(t)\iota(t)]\}dt \tag{A1}$$

subject to the state equation $\dot{\Phi}=\iota(t)$, $\dot{d}=[(\delta+\delta_A)s/\gamma]+(\mu-\mu_A)\iota(t)d(t)-nd(t)$, the initial conditions $\Phi(0)=0$, $d(0)=d_0>0$, and the control constraint, $(1-s)/a_\iota d(t)\geq\iota(t)\geq 0$ for all t. The current value Hamiltonian for this problem can stated as:

$$H=\Phi\log\lambda+\log[1-s-a_\iota d\iota]+\theta\iota+\eta[(\delta+\delta_A)s/\gamma+(\mu-\mu_A)\iota d-nd].$$

For an interior solution, the first order condition is:

$$\frac{\partial H}{\partial \iota}=-\frac{a_\iota d}{(1-s-a_\iota d\iota)}+\theta+\eta(\mu-\mu_A)d=0\,. \tag{A2}$$

The costate equations are

$$\dot{\theta} = (\rho - n)\theta - \frac{\partial H}{\partial \Phi} = (\rho - n)\theta - \log \lambda \tag{A3}$$

$$\dot{\eta} = (\rho - n)\eta - \frac{\partial H}{\partial d} = (\rho - n)\eta + \frac{a_\iota \iota}{1 - s - a_\iota d\iota} - \eta\big[(\mu - \mu_A)\iota - n\big]. \tag{A4}$$

I solve the optimal control problem for a balanced growth equilibrium in which all endogenous variables grow at constant rates (although not necessarily the same rate) and investment in R&D is positive $\iota > 0$. It follows from (7) that for $\dot{D}/D$ to be constant, $X(t)/D(t)$ and ι must be constant as well. Given $\dot{X}/X = n$ by (21), for $X(t)/D(t)$ to be constant, $\dot{D}/D = n$ must hold. This in turn implies that $d = D(t)/N(t)$ is constant. Substituting $R(t) = \iota D(t)$ into (20) implies that c must be a constant. Let $g_\theta = \dot{\theta}/\theta$. Constancy of g_θ requires θ be a constant, which in turn implies $\dot{\theta}/\theta = 0$. Let $g_\eta = \dot{\eta}/\eta$. With ι and d constant, constancy of g_η requires η be a constant, which in turn implies $\dot{\eta}/\eta = 0$.

Imposing $\dot{\theta} = 0$ and $\dot{\eta} = 0$ on (A3) and (A4) implies $\theta = \log\lambda/(\rho - n)$ and $\eta = a_\iota \iota/[1 - s - a_\iota d\iota)(\iota(\mu - \mu_A) - \rho)]$. Imposing $\dot{d} = 0$ yields $d = (\delta + \delta_A)s/[\gamma(n - \iota(\mu - \mu_A))]$. Note that (20) collapses to $c/\lambda = 1 - s - a_\iota d\iota$. Substituting for θ, η and d into (A2) using $c/\lambda = 1 - s - a_\iota d\iota$ gives (31). The RD^{SO} and LM conditions, given by (31) and (27) respectively, determine the optimal balanced growth levels $\tilde{\iota}$ and $\tilde{c}$. To see uniqueness, consider plotting the RD^{SO} and LM curves in (ι, c) space restricting the domain to $\lambda(1 - s) > c > 0$ and $n/(\mu - \mu_A) > \iota > 0$. For the RD^{SO} equation: $(dc/d\iota)\big/_{RD^{SO}} > 0$. Moreover, as $\iota \to 0$, $c \to (c_0)^{SO} = \lambda s a_\iota (\rho - n)(\delta + \delta_A)/(\log\lambda) n\gamma$, and as $\iota \to \iota^{max} = n/(\mu - \mu_A)$, $c \to \infty$. For the LM equation: $(dc/d\iota)\big/_{LM} > 0$. Furthermore, as $\iota \to 0$, $c \to \lambda(1 - s)$, and as $\iota \to \iota^{max} = n/(\mu - \mu_A)$, $c \to -\infty$. Hence, for a unique equilibrium, the intercept of the LM curve must be strictly higher than that of the RD^{CE} curve: $\lambda(1 - s) > (c_0)^{SO} \Rightarrow \log\lambda > [s a_\iota (\rho - n)(\delta + \delta_A)] / [(1 - s)n\gamma]$. Observe that this is quite similar to the uniqueness condition for competitive equilibrium which was $\lambda(1 - s) > c_0 \Rightarrow \lambda - 1 > [s a_\iota (1 - \phi_\iota)(\rho - n)(\delta + \delta_A)]/[(1 - s)n\gamma]$.

To analyze stability, I invoke the transversality conditions:

$$\lim_{t\to\infty} e^{-(\rho-n)t}\theta(t)\Phi(t) = 0.$$

$$\lim_{t\to\infty} e^{-(\rho-n)t}\ \eta(t)d(t) = 0.$$

The transversality condition for θ and the costate equation imply that $\theta(t) = log\lambda(\rho - n)$ for all $t > 0$ [see Grossman and Helpman (1991), p. 71 and 103]. To see this formally, note that the solution for $\dot{\theta} = (\rho - n)\theta - log\,\lambda$ is $\theta(t) = \theta_0 e^{(\rho-n)t} - [\,log\lambda\,(e^{(\rho-n)t} - 1)]/\,(\rho - n)$, where θ_0 is the initial value for θ, which I assume exists but is unknown. Substituting this into the transversality condition implies $\lim_{t\to\infty} e^{-(\rho-n)t}\theta(t)\Phi(t) = \{\theta_0 - [log\,\lambda/(\rho - n)]\}\ \Phi(t) = 0$. Given $\lim_{t\to\infty} \Phi(t) = \infty$, the only possible solution to this limit is $\theta_0 = log\,\lambda/(\rho - n)$. The solution to $\dot{\theta}$ then implies

$$\theta(t) = \frac{log\,\lambda}{\rho - n}$$

Substituting this into the transversality condition and using the L'Hopital's rule implies that $\lim_{t\to\infty} e^{-(\rho-n)t}\theta(t)\Phi(t) = [log\,\lambda/(\rho - n)]\,\iota(t)/\infty$, which equals zero for a finite level of $\iota(t)$ which follows from the constraint $(1 - s)/a_\iota d(t) \geq \iota(t) \geq 0$.

Substituting for $\theta(t) = log\,\lambda/(\rho - n)$ into (A2) gives:

$$\iota = \iota(\eta, d) = \frac{1}{d}\left[\frac{1-s}{a_\iota} - \frac{1}{log\,\lambda/[\,d(\rho - n)] + \eta(\mu - \mu_A)}\right],$$

where $\partial\iota(\eta,d)/\partial\eta > 0,\ \partial\iota(\eta,d)/\partial d < 0$. Substituting $\iota(\eta,d)$ and $\theta(t)$ into the costate equation for η and the equation of motion for d implies the following autonomous system:

$$\dot{\eta} = \eta[\,\rho - (\mu - \mu_A)\iota(\eta, d)] + \frac{a_\iota}{[(1-s)/\iota(\eta,d)] - a_\iota d}$$

$$\dot{d} = \frac{(\delta + \delta_A)s}{\gamma} + [(\mu - \mu_A)\iota(\eta, d) - n]\,d\,.$$

At the steady-state for $\tilde{d} > 0$, we need $n - \iota(\mu - \mu_A) > 0$. So, we restrict attention the neighborhood of the steady-state where $\iota < n/(\mu - \mu_A)$ holds. Observe that $d\dot{\eta}/d\eta$ and thus $(dd/d\eta)\big|_{\dot{\eta}=0}$ cannot be signed unambiguously. On the other hand, around the steady-state $d\dot{d}/dd < 0$ and $d\dot{d}/d\eta > 0$; thus, $(dd/d\eta)\big|_{\dot{d}=0} > 0$. In other words, $\dot{d}=0$ is upward sloping, and starting from $\dot{d}=0$ an increase (decrease) in d renders $\dot{d} < 0$, causing d to decrease (increase). Given that a unique steady-state equilibrium exists, we can examine all of the possibilities. First, when $\dot{\eta} = 0$ is upward sloping, $\dot{\eta} = 0$ may be steeper of flatter than $\dot{d} = 0$ and $d\dot{\eta}/d\eta$ maybe positive or negative. Hence, with $\dot{\eta}=0$ upward sloping, there are four possibilities. It can be shown graphically that the system is saddle path stable in either case. Second, when $\dot{\eta} = 0$ is downward sloping, $d\dot{\eta}/d\eta$ may be positive or negative. Thus, with $\dot{\eta} = 0$ there are two possibilities. It can be shown graphically that the system is saddle-path stable when $d\dot{\eta}/d\eta > 0$ and exhibits a stable focus when $d\dot{\eta}/d\eta < 0$.

Appendix C: Identifying the Marginal Impact of Innovation

I now derive an expression for the welfare impact of a marginal innovation, following closely Grossman and Helpman (1991, pp.110-111) and Segerstrom (1998, pp. 1308-1309). I consider a situation in which an external agent becomes successful in innovating a higher quality product in industry ω at time $t = 0$. I then investigate the impact of this event on the welfare of all economic agents other than the external agent. To do this, I perturb the competitive equilibrium solution by $d\Phi$ at time $t = 0$ and investigate the impact on the discounted welfare for the period $(0, \infty)$. I exclude the welfare of the external agent from the analysis because the free-entry in R&D condition implies that for any entrepreneur engaged in R&D, the R&D costs must be exactly balanced by the expected discounted rewards from R&D. Note that with the measure one of structurally-identical industries, it again follows that $D(\omega,t) = D(t)$, $\iota(\omega,t) = \iota(t)$, $X(\omega,t) = X(t)$ for all ω and t.

Let $E(t) = ce^{nt}$ denote the aggregate consumer expenditure. Thus, (30) can be restated as:

$$U = \int_0^\infty e^{-(\rho-n)t}\left[\Phi(t)\log\lambda + \log\frac{E(t)}{e^{nt}\lambda}\right]dt$$

To derive the welfare impact of an incremental innovation, I differentiate the above:

$$\frac{dU}{d\Phi} = \int_0^\infty e^{-(\rho-n)t}\log\lambda dt + \int_0^\infty e^{-(\rho-n)t}\frac{1}{E(t)}\frac{dE(t)}{d\Phi(t)}dt\ .$$

The first term equals $\log\lambda/(\rho-n)$ and capture the *consumer surplus externality*. The second term captures the *business stealing* and *intertemporal R&D spillover externalities*. The successful innovation by the external agent leads to the replacement of the incumbent firm in industry ω, resulting in a loss of stream of monopoly profits and lower incomes for its stockholders. In addition, the marginal innovation raises the difficulty of future research, resulting in more resources being diverted to innovation activities. Both of these effects lead to lower consumption expenditure, which are compounded by multiplier effects.

I now explicitly identify the business stealing and intertemporal R&D externalities. Note that aggregate expenditure equals aggregate income minus aggregate savings:

$$E(t) = [sw + (1-s)]e^{nt} + \pi(t) - \iota a_\iota D(t),$$

where the first term measures the labor income from specialized and non-specialized labor, the second term measures the aggregate profit income (excluding that of the external agent), and the third term measures the aggregate investment in R&D. Differentiating $E(t)$ with respect to Φ and noting $d\Phi/d\iota = 0$ gives:

$$\frac{dE(t)}{d\Phi(t)} = se^{nt}\frac{dw}{d\Phi} + \frac{d\pi}{d\Phi} - \iota a_\iota\frac{dD(t)}{d\Phi}\ .$$

Recall that by combining (13) and (19), which hold both in and out of the steady-state, we have derived $w = a_\iota\delta(1-\phi_\iota)\iota/\gamma(1+\mu\iota)$ thus $dw/d\Phi = 0$. Next, I consider $d\pi/d\Phi$. This term captures the decline in aggregate expenditure associated with the loss of profits in industry ω due to the marginal innovation. In this model, the effective replacement rate takes into account the rent protection costs of incumbent firms and equals $\iota(1+\eta(\iota))$. Since the arrival of innovations is governed by a Poisson process whose intensity equals ι, the arrival of effective replacement incidents is also governed by a Poisson process whose intensity equals $\iota(1+\eta(\iota))$. It follows from the properties of the Poisson process that the effective duration of monopoly power is exponentially distributed with parameter $\iota(1+\eta(\iota))$. In the event of no further innovation between time 0 and time t— that is, between the time the external agent innovates and the time that signifies the current period—with probability $e^{-\iota(1+\eta(\iota))t}$, no further innovation occurs and the economy forfeits monopoly profits at time t of amount $(\lambda-1)E(t)/\lambda$. In addition there is a multiplier effect because the loss of profits in industry ω will induce a fall in aggregate expenditure, which will translate into lower incomes and expenditures in other industries. The combined change in total profits equals:

$$\frac{d\pi}{d\Phi}=-\frac{\lambda-1}{\lambda}E(t)e^{-\iota(1+\eta(\iota))}+\frac{\lambda-1}{\lambda}\frac{dE}{d\Phi}.$$

Next, consider $\iota a_{\iota}\dfrac{d(D(t))}{d\Phi}$, which captures the expenditure decline associated with the higher resource requirement in R&D due to marginal innovation. The first step is to find the solution to the differential equation (7). Note that $D(\omega,t)=D(t)$, $\iota(\omega,t)=\iota(t)$ and $X(\omega,t)=X(t)$ for all ω and t. In addition, $X_A(t)=X(t)$ and $\iota_A(t)=\iota(t)$. The solution to (7) is then given by:

$$D(t)=D_0e^{\Phi(t)(\mu-\mu_A)}+\frac{(\delta+\delta_A)s}{\gamma}e^{\Phi(t)(\mu-\mu_A)}\int_0^t e^{ns-(\mu-\mu_A)\Phi(s)}ds,$$ where D_0 is the level of R&D difficulty at time 0. Differentiating this gives:

$$\frac{dD(t)}{d\Phi}=(\mu-\mu_A)D_0e^{(\mu-\mu_A)\Phi(t)}.$$

Substituting the expressions for $d\pi/d\Phi$ and $dD(t)/d\Phi$ into the $dE(t)/d\Phi$ expression and collecting terms gives:

$$\frac{dE(t)}{d\Phi(t)}\frac{1}{E(t)} = -(\lambda - 1)e^{-\iota(1+\eta(\iota))} - \frac{1}{E}\lambda\iota a_{\iota}(\mu - \mu_A)D_0 e^{(\mu-\mu_A)\Phi(t)} .$$

Substituting the above expression now into $dU/d\Phi$ using $E = [\lambda a_{\iota} D(t)(\rho + \iota(1 + \eta(\iota)) - n)]/(\lambda - 1)$, which follows from (25) and (13), and calculating the integral implies:

$$\frac{dU}{d\Phi} = \frac{\log\lambda}{\rho - n} - \frac{\lambda - 1}{\rho + \iota(1+\eta(\iota)) - n} - \frac{\iota(\lambda - 1)(\mu - \mu_A)}{\rho + \iota(1+\eta(\iota)) - n}\int_0^{\infty}\frac{D_0 e^{-(\rho-n)t+(\mu-\mu_A)\Phi(t)}}{D(t)}dt$$

To evaluate the integral term, I invoke the steady-state properties of the model as in Segerstrom (1998, p. 1309). At the steady-state $\iota(t) = \iota$, $\Phi(t) = \iota t$ and $D(t) = D_0 e^{nt}$. Substituting these into the integral term and evaluating gives (33).

Appendix E: Effects of parameter changes on R&D policy

We can use the simulation results to determine each parameter's qualitative impact on spillovers that works indirectly by altering the innovation rate ι. Recall that these were ambiguous and involved multiple effects. The one analytical result we had was that an increase in ι decreases the BS effect. The simulations resolve the ambiguity on the IS effect, revealing that an increase in ι decreases the IS effect [recall that there were two competing forces on IS: a higher ι reduces the BS component of the IS effect and at the same time it increases the spillover factor in the IS effect].[32] Thus, I can state the following

Lemma 3: *Numerical simulations imply that an increase in ι reduces the negative externalities BS and IS and thereby calls for a rise in ϕ_{ι}^{SO}. Thus, any parameter change that leads to a larger innovation rate ι pushes the R&D policy toward subsidy.*

Armed with Lemma 3 we can now shed further light on the simulations using the results from Proposition 1 and the analytical exercise in the previous section.

E.1. An increase in λ

[32] The derivations are available from the author upon request. The results hold for a wide range of parameters.

A higher λ leads to an increase in ι (Proposition 1), which indirectly reduces the size of the negative externalities (Lemma 3) and thus calls for an increase in ϕ_ι^{SO}. These must be weighed against the direct effects though. A larger λ increases both the CS effect (a positive externality) and the BS effect (a negative externality) and at the same exerts an ambiguous effect on IS effect (a negative externality). Simulations imply that the welfare gains are negative when λ is of small and large magnitude, and positive when λ is of medium magnitude.

E. 2. An increase in μ

A higher μ leads to an decrease in ι (Proposition 1), which indirectly increases the size of the negative externalities (Lemma 3) and thus calls for reduced R&D subsidy. In addition, there are the direct effects. A larger μ increases the BS effect and the IS effect, both of which are negative externalities. Hence the direct effects also call for reduced R&D subsidies. Therefore we can conclude that an increase in μ would imply a fall in ϕ_ι^{SO} .

D.3. An increase in μ_A

A higher μ_A leads to an increase in ι (Proposition 1), which indirectly reduces the size of the negative externalities (Lemma 3) and thus calls for an increase in R&D subsidy. In addition, the direct effects complement this impact. A larger μ_A decreases the BS and IS effects, both of which are negative externalities. Therefore we can conclude that an increase in μ would imply an increase in ϕ_ι^{SO}.

E.4. An increase in δ_A

A higher δ_A leads to an decrease in ι (Proposition 1), which indirectly increases the size of the negative externalities (Lemma 3) and thus calls for a reduction in R&D subsidy. Again there are also the direct effects that reinforce this mechanism. A larger δ_A increases the magnitudes of negative externalities the BS and IS effects. Therefore we can conclude that an increase in δ_A would imply a fall in the optimal R&D subsidy rate. The effects for δ are the same, simply working in the opposite direction.

It is straightforward to determine the optimal R&D policy effects associated with the rest of the parameters using Proposition 1, Lemma 3 and (33). Table 1 and Figure 2 summarize the resulting outcomes.