Referee's Appendices[*]

for

Intellectual Property Rights and Rent Protection in a North-South Product-Cycle Model

by

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[*]Not to be considered for publication. To be made available on the author's web site and also upon request from the author.

Appendix A: Existence, Uniqueness and Other Technical Details of Model

(Not to be considered for publication, to be made available upon request and also on my web site)

1. Existence and Uniqueness of the Steady-State Equilibrium

Step 1: Substituting for $c(\iota,\mu)$ and $w_{LN}(\iota,\mu)$ from (31) and (32) into the *Southern* general purpose labor market condition (34) using (24) gives:

$$\frac{\mu}{\iota+\mu}\left[A_\mu s_S\eta_S\left(D'(\rho+\iota-n)+\iota\right)+E's_N(\rho+2\iota+2\mu-n)\right]=(1-s_S)\eta_S \tag{A.1}$$

where $D' \equiv (1-\sigma_\mu)/(\lambda-1)$ and $E' \equiv A_\iota(1-\sigma_\iota)\lambda/(\lambda-1)$. In brackets, the term $A_\mu s_S\eta_S\iota$ captures the labor demand from imitative activity, whereas the rest of the terms capture the labor demand from Southern manufacturing. Substituting for $c(\iota,\mu)$ from (31) into the *Northern* general purpose labor market condition (33) using (24) gives:

$$\iota\left[\frac{A_\mu s_S\eta_S D'(\rho+\iota-n)}{(\iota+\mu)}+A_\iota s_N\left(\frac{E(\rho+2\iota+2\mu-n)}{(\iota+\mu)}+1\right)\right]=1-s_N\ , \tag{A.2}$$

where $E \equiv (1-\sigma_\iota)/(\lambda-1)$. In brackets, the term $A_\iota s_N\iota$ captures the labor demand from innovative activity, and the rest of the terms capture the demand from Northern manufacturing.

Step 2: I now illustrate the steady-state equilibrium on $(\mu,\ \iota)$ space, focusing on the strictly positive range for both variables. First, I solve (A.1) for ι. This yields:

$$\iota=\frac{(1-s_S)\eta_S-2E's_N\mu-(\rho-n)(A_\mu s_S\eta_S D'+E's_N)}{A_\mu s_S\eta_S(1+D')+2E's_N-((1-s_S)\eta_S/\mu)}\ . \tag{SL′}$$

I consider the case in which both the numerator and the denominator in (SL′) are strictly positive.[1] This requires $\mu^L \equiv (1-s_S)\eta_S/[A_\mu s_S\eta_S(1+D')+2E's_N] < \mu < (1-s_S)\eta_S-(\rho-n)(A_\mu s_S\eta_S D'+E's_N)/2E's_N \equiv \mu^U$. Within this interval for μ, it follows that $(d\iota/d\mu)\,/_{SL'} < 0$. The (SL′) equation implies that as $\mu \to \mu^L$, $\iota \to \infty$; and as $\mu \to \mu^U$, $\iota \to 0$. Observe that for $\mu^U > \mu^L$, we need $(1-s_S)\eta_S A_\mu s_S\eta_S(1+D') > (\rho-n)(A_\mu s_S\eta_S D' + E's_N)[A_\mu s_S\eta_S(1+D')+2E's_N]$; for $\mu^L > 0$, we need $(1-s_S)\eta_S > (\rho-n)(A_\mu s_S\eta_S D'+E's_N)$. Note that both inequalities hold if $\rho-n$ is sufficiently small.

Similarly, I solve (A.2) for μ. This gives:

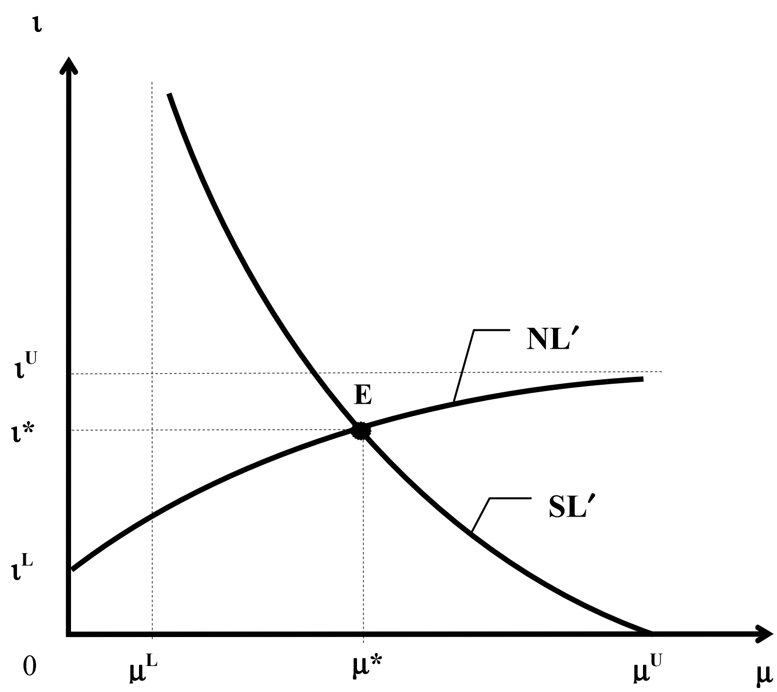
$$\mu=\frac{\iota[A_\mu s_S\eta_S D'+A_\iota s_N(1+2E)]+(A_\mu s_S\eta_S D'+A_\iota s_N E)(\rho-n)-(1-s_N)}{[(1-s_N)/\iota]-A_\iota s_N(1+2E)}\ . \tag{NL′}$$

[1] Focusing on the case in which both the numerator and denominator are negative is not compatible with an equilibrium that allows for a positive level employment in *imitative* activity. To see this, first note that the model must have a solution for a negligible discount rate, i.e., when $(\rho-n) \to 0$. In this case, a strictly positive level of employment in imitative activity requires $(1-s_S)\eta_S - 2E's_N\mu - [A_\mu s_S\eta_S D'\iota\mu/(\iota+\mu)] > 0$ (See A.1). Thus $(1-s_S)\eta_S - 2E's_N\mu > 0$ must hold. This in turn implies that in (SL′) the numerator and hence the denominator must be positive.

I again consider the case in which both the numerator and the denominator in (NL′) are strictly positive.[2] This requires $\iota^L = [(1 - s_N) - A_\mu s_S \eta_S D' - A_\iota s_N E]/[A_\mu s_S \eta_S D' + A_\iota s_N (1 + 2E)] < \iota < (1 - s_N)/A_\iota s_N(1 + 2E) = \iota^U$. Within this interval for ι, it follows that $(d\iota/d\mu) \big/_{NL'} > 0$. The (NL′) equation implies that as $\iota \rightarrow \iota^L$, $\mu \rightarrow 0$; and as $\iota \rightarrow \iota^U$, $\mu \rightarrow \infty$. Observe that $\iota^U > \iota^L$ always holds. For $\iota^L > 0$, we need $(1 - s_N) - A_\mu s_S \eta_S D' - A_\iota s_N E > 0$.

The above analysis implies that to establish a unique equilibrium in the strictly positive orthant, we need two assumptions: $(\rho - n)$ being sufficiently small and $(1 - s_N) - A_\mu s_S \eta_S D' - A_\iota s_N E > 0$. Figure A1 illustrates the unique steady-state equilibrium. Using the other steady-state relationships, it is straightforward to obtain strictly positive equilibrium values for the rest of the endogenous variables. With $\iota > 0$ and $\mu > 0$, (31) implies that $c > 0$ and (32) implies that $\lambda > w_{LN} > 1$. These along with (27) and (28) imply that $w_{HS} > 0$ and $w_{HN} > 0$.

Figure A1: The steady-state equilibrium



[2] Focusing on the case in which both the numerator and denominator are negative is not compatible with an equilibrium that allows for a positive level of employment in *innovative* activity. To see this, we again consider the case when $(\rho - n) \rightarrow 0$. In this case, a strictly positive level for $A_\mu s_S \eta_S D' \iota/(\iota + \mu)$ requires $[(1 - s_N)/\iota] - A_\iota s_N(1 + 2E) > 0$ (See A.2). This in turn implies that in (NL′) the denominator and hence the numerator must be positive.

2. Second Order Conditions for Optimal Rent Protection Activities

Consider the discussion in the derivation of optimal rent protection activities. Note that the incremental gain in the rate of return from undertaking one more unit of innovation-deterring activity is ι/X_ι, which is monotonically decreasing in X_ι. Moreover as $X_\iota \to 0$, $\iota/X_\iota \to \infty$ and as $X_\iota \to \infty$, $\iota/X_\iota \to 0$. On the other hand, the marginal cost of one more unit of innovation-deterring activity per firm value is $w_{HN}\gamma_\iota/\upsilon_N$, which is independent of X_ι. Thus, marginal profitability of innovation-deterring activity is globally decreasing in X_ι ; consequently, the second order condition for optimality is satisfied. The same argument can be used to show that second order condition holds for the optimal level of imitation-deterring activity.

Appendix B: Choice of Benchmark Parameters

(Not to be considered for publication, to be made available upon request and also on my web site)

For simulating the Model with FDI (Extension A), I chose the following benchmark parameters:

$\lambda = 1.25$, $\rho = 0.07$, $n = 0.014$, $L_N = 1$, $L_S = 2$, $\alpha = 1.05$, $s_S = 0.02$, $s_N = 0.04$,

$a_\iota = 0.9$, $a_{\mu N} = 16$, $a_{\mu F} = 8$, $a_\phi = 0.6$, $\gamma_\iota = 1$, $\gamma_{\mu N} = 8$, $\gamma_{\mu F} = 8$, $\delta_\iota = 1$, $\delta_{\mu N} = 1$, $\delta_{\mu F} = 2$.

The size of innovations, λ, measures the gross mark up (the ratio of the price to the marginal cost) enjoyed by innovators and is estimated as ranging between 1.05 and 1.4 [see Basu, 1996, and Norrbin, 1993]. The world population growth rate, n, is calculated as the annual rate of world population growth between 1991 and 2000 according to the World Development Indicators (World Bank, 2003). The subjective discount rate, ρ, is set at 0.07 to reflect a real interest rate of 7 percent, consistent with the average real return on the US stock market over the past century as calculated by Mehra and Prescott (1985). The ratio of Southern population to Northern population $\eta_S = L_S/L_N$ is set at *2*, calculated as the ratio of the working age population in middle income countries to that in high income countries—as defined by the World Bank (World Development Indicators, 2003).

The marginal cost of multinationals relative to Southern firms α is set at 1.05, which is at one fifth of the distance between the numeraire and λ. The percentage of specialized labor s_N and s_S are set at 0.04 and 0.02 respectively. It is reasonable to think that this type of labor constitutes a small fraction of the aggregate labor force, and that rent protection is more institutionalized in the North. I set $a_{\mu N} > a_{\mu F} = 8$ to capture the fact that targeting a multinational for imitation should require fewer resources than targeting a Northern firm. I set $\delta_{\mu F} > \delta_{\mu N}$ to capture the fact that Multinationals can deter imitation compared to Northern firms.

The rest of the benchmark parameters are chosen with three objectives in mind: *i)* To generate a growth rate $g = \iota \log\lambda$ in the neighborhood of 0.5 percent and a wage rate w_{LN} greater than one [the motivation to target $g = 0.5$ comes from Denison (1985) and Jones (2002), who suggest that the rate of growth driven by knowledge advancements is in the neighborhood of 0.5 percent] *ii)* To obtain positive levels for endogenous variables that are consistent with common sense and with the empirical literature/available data *iii)* To get a ranking for stock market valuations such that the multinationals realize the highest stock market valuations and the highest profit flows in relation to other firm types (more specifically, $v_F > v_{SN} > v_N > v_{SF}$ and $\pi_F > \pi_{SN} > \pi_N > \pi_{SF}$). In general, the benchmark outcomes are in line with the recent relevant papers [see Lundborg and Segerstrom 1999; Sayek and Sener 2006].

In the FDI model with endogenous β, I assume the following functional form $a_\phi = A_\phi[\beta/(1-\beta)]^\psi$, where $A_\phi > 0$ and $\psi > 1$. These restriction on the parameters together with the functional form choice guarantee that $a_\phi' > 0$ and $a_\phi'' > 0$ and there exists an interior solution for β in the interval $(0,1)$.

References

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Appendix C: The Imitation Model with Homogenous Labor

(not to be considered for publication, to be made available from the author upon request and also on my web site)

I modify the imitation model by assuming one type of labor in both the North and South—instead of differentiating labor as specialized and non-specialized. In the North, labor is mobile between innovation, manufacturing and innovation-deterring activities. In the South, labor is mobile between imitation, manufacturing and imitation-deterring activities. The resulting model was sufficiently complicated to obtain analytical result. Therefore, I ran numerical simulations of the model using Mathematica Version 5. It is straightforward to derive the steady-state equations for this model, which are presented in the attached Mathematica program.

Choice of Benchmark Parameters

For the simulations, I chose the following benchmark parameters.

$\lambda = 1.25$, $n = 0.014$, $\rho = 0.07$, $\eta_S = 2.0$, $A_\mu = 1000$, $A_\iota = 130$.

The choice of λ, n, ρ, and η_S follow from Appendix B. A_μ and A_ι are chosen with two objectives in mind: i) to generate a growth rate $g = \iota log\lambda$ in the neighborhood of 0.5 percent and a North-South relative wage rate w_N greater than one; ii) to generate positive levels for endogenous variables that are consistent with common sense and with the empirical literature/available data.

See the attached Mathematica program for steady-state equations and numerical simulation results.

Discussion of Results and Robustness Checks

Note that the benchmark simulations imply that An increase in A_μ leads to $dw_N > 0$, $d\iota < 0$, $d\mu < 0$, $dm < 0$, $dn_N > 0$. The findings on A_μ are identical to those in Proposition 1. These results continue to hold when simulations are rerun for high and low values for parameters as indicated in the table below.[1]

An increase in A_μ			
Parameters	**Benchmark**	**Low**	**High**
A_ι	130	61	163
A_μ	1000	200	5000
η_S	2.0	1.7	10
λ	1.25	1.05	1.4
ρ	0.07	0.014	0.084

[1] For an increase in A_μ, it is worth to note the following. Beginning from the benchmark parameter values, if one sets $A_\iota \geq 164$ or $\eta_S \leq 1.6$ or $\rho \geq 0.085$, then the benchmark results change as $dw_N > 0$, $d\iota > 0$, $d\mu > 0$, $dm > 0$, $dn_N < 0$. Observe that in all of these cases, the results that differ from those in Prop 1. are $d\iota > 0$, $d\mu > 0$ and $dm > 0$. Also, for an increase in η_S, it is worth to note the following. If $\eta_S \geq 4.3$, then the benchmark results change as $dw_N < 0$, $d\iota > 0$, $d\mu > 0$, $dm > 0$, $dn_N < 0$. The only result that differs from the benchmark is $dw_N < 0$.

```
"APPEDNDIX C: MATHEMATICA PROGRAM FOR THE MODEL WITH IMITATION AND HOMOGENOUS LABOR";

"- This program requires Mathematica Version 5.0.
 - I drop the disctinction between specialized and general purpose labor
  and assume that there is only one type of labor in both the North and the
  South.\n- Notes: For convenience, the subscripts and superscripts of the
  main model are entered in regular format. The parameter 'ng' stands for
  the rate of population growth, 'wN' stands for the Northern wage rate
  relative to Southern wage rate, 'i'stands for  the rate of innovation.";

"THE BENCHMARK PARAMETERS";

"First, I clear the parameters and variables"; Clear[ρ, ng, σμ, σi, i, μ, Aμ, Ai, λ, ηS];

"Benchmark parameters are as follows";

ρ = 0.07; ng = 0.014; σμ = 0; σi = 0; Aμ = 1000; Ai = 130; λ = 1.25; ηS = 2.0;

"THE MAIN EQUATIONS OF THE MODEL";

"-The following describe the steady-state equilibrium
   as a sytem of five equation in five unknowns: wN,i,μ,m and nN";
```

$$EQ1 = \frac{(wN - 1)}{wN * ((\rho - ng) + i)} - \frac{(\lambda - wN) * \mu * (i / (i + \mu)) * A\mu * (1 - \sigma\mu)}{\lambda((\rho - ng) + 2i + 2\mu)};$$

$$EQ2 = \frac{\mu}{(i * wN)} + \frac{\mu * (\lambda - wN) * ((A\mu * \mu * (i / i + \mu)) + 1)}{\lambda * ((\rho - ng) + 2i + 2\mu)} - \eta S * \left(\frac{1}{\lambda} + \frac{i * (\lambda - wN) * ((i * Ai) + 1)}{wN * \lambda * ((\rho - ng) + 2i + 2\mu)}\right);$$

```
EQ3 = μ - i * ((i * Ai * (1 - σi)) - 1);
EQ4 = m - (μ * i / (i + μ));
EQ5 = nN - (i / (i + μ));
FindRoot[{EQ2 == 0, EQ1 == 0 , EQ3 == 0, EQ4 == 0, EQ5 == 0}, {wN, 1.22},
 {i, 0.02}, {μ, 0.03}, {m, 0.011}, {nN, 0.3}, MaxIterations → 500]

{wN → 1.21314, i → 0.0206916, μ → 0.0349671, m → 0.0129993, nN → 0.371759}

Cikti[1] = %;

x = {wN, i, μ, m, nN} /. %; Positive[x]

{True, True, True, True, True}
```

```
g = 0.0206 * Log[λ]
```

```
0.00459676
```

```
"EFFECTS OF STRENGTHENING IPRS PROTECTION:
 A 20% increase in Aμ";
```

```
Clear[ρ, ng, σμ, σi, i, μ, Aμ, Ai, λ, ηS]; Clear[EQ1, EQ2, EQ3, EQ4, EQ5];
```

```
ρ = 0.07; ng = 0.014; σμ = 0; σi = 0; Aμ = 1000; Ai = 130; λ = 1.25; ηS = 2.0; Aμ = 1000 * (1.20);
```

$$EQ1 = \frac{(wN - 1)}{wN * ((\rho - ng) + i)} - \frac{(\lambda - wN) * \mu * (i / (i + \mu)) * A\mu * (1 - \sigma\mu)}{\lambda ((\rho - ng) + 2 i + 2 \mu)};$$

$$EQ2 = \frac{\mu}{(i * wN)} + \frac{\mu * (\lambda - wN) * ((A\mu * \mu * (i / i + \mu)) + 1)}{\lambda * ((\rho - ng) + 2 i + 2 \mu)} - \eta S * \left(\frac{1}{\lambda} + \frac{i * (\lambda - wN) * ((i * Ai) + 1)}{wN * \lambda * ((\rho - ng) + 2 i + 2 \mu)}\right);$$

```
EQ3 = μ - i * ((i * Ai * (1 - σi)) - 1); EQ4 = m - (μ * i / (i + μ)); EQ5 = nN - (i / (i + μ));
FindRoot[{EQ2 == 0, EQ1 == 0 , EQ3 == 0, EQ4 == 0, EQ5 == 0}, {wN, 1.22},
 {i, 0.02}, {μ, 0.03}, {m, 0.011}, {nN, 0.3}, MaxIterations → 500]
```

```
{wN → 1.21863, i → 0.0206853, μ → 0.0349395, m → 0.012993, nN → 0.371873}
```

```
dwN = (wN /. %) - (wN /. Cikti[1]); di = (i /. %) - (i /. Cikti[1]); dμ = (μ /. %) - (μ /. Cikti[1]);
dm = (m /. %) - (m /. Cikti[1]); dnN = (nN /. %) - (nN /. Cikti[1]);
{{"dwN" → dwN}, {"di" → di}, {"dμ" → dμ}, {"dm" → dm}, {"dnN" → dnN}}
```

$$\{\{dwN \to 0.00548893\}, \{di \to -6.3078 \times 10^{-6}\}, \{d\mu \to -0.0000276219\}, \{dm \to -6.3078 \times 10^{-6}\}, \{dnN \to 0.000113365\}\}$$

```
"Observe that all qualitative changes conform with Proposition 1. Hence,
   one can add the following line and check the robustness of the results by
   re-running the simulations with high and low values for the parameters.";
```

```
If[dwN > 0 && di < 0 && dμ < 0 && dm  < 0 && dnN > 0, OK, NO]
```

```
OK
```

```
"APPEDNDIX D: MATHEMATICA PROGRAM FOR THE MODEL WITH IMITATION";
```

```
"- This program requires Mathematica Version 5.0. Before evaluating the cells, '
  Math Econ' package written by Cliff Huang and Philip Crooke needs to be
  run. This package accompanies the book 'Mathematics and Mathematica for
  Economists', 1997, Blackwell Publishers: Oxford, written by the above authors.
- Objective: To conduct comparative steady-state analysis
- Notes: In this program, for convenience I enter the subscripts and superscripts of
  the main model in regular format,\n- Slight change in notation: The parameter 'ng'
  stands for the rate of population growth. 'σi' stands for (1 - the subsidy rate for
  innovation) and 'σμ' stands for (1 - the subsidy rate for imitation). Define dr = ρ-
  n. The elimination of minus terms help Mathematica to obtain more tidy expressions.";
```

```
"THE MAIN EQUATIONS OF THE MODEL";
```

```
"First, I clear the variables";

Clear[FEIN, FEIM, GPNO, GPSO, F1, F2, F3, F4];

Clear[c, i, μ, wLN];

Clear[ηS, λ, dr, ρ, n, σi, σμ, Ai, Aμ, sS, sN];

"I enter the Free-Entry in Innovation and Imitation Conditions";
```

$$FEIN = \frac{c\,(1+\eta S)\left(1-\frac{wLN}{\lambda}\right)}{dr+2\,i+2\,\mu} == Ai*(\sigma i)*sN*wLN;$$

$$FEIM = \frac{c\,(1+\eta S)\left(1-\frac{1}{wLN}\right)}{dr+i} == A\mu*(\sigma\mu)*sS*\eta S;$$

```
"I solve FEIN and FEIM for c and wLN";
```

```
Solve[{FEIN, FEIM}, {c, wLN}] // FullSimplify
```

$$\left\{\left\{c \to \frac{\lambda\,(Ai\,sN\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu)}{(1+\eta S)\,(-1+\lambda)},\ wLN \to \frac{\lambda\,(Ai\,sN\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu)}{Ai\,sN\,\lambda\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu}\right\}\right\}$$

```
"I define closed form solutions for c and wLN in terms of i and μ using the above";
```

$$\{c, wLN\} = \left\{\frac{\lambda\,(Ai\,sN\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu)}{(1+\eta S)\,(-1+\lambda)},\ \frac{\lambda\,(Ai\,sN\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu)}{Ai\,sN\,\lambda\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu}\right\};$$

"I define nN,nS and m as follows"; $nN = \frac{i}{(i+\mu)}$; $nS = \frac{\mu}{(i+\mu)}$; $m = \mu * nN$;

```
"The general purpose labor market equilibrium conditions for the North and South are";
```

$$GPNO = \left(\frac{i}{i+\mu}\right)\left(\frac{c\,(1+\eta S)}{\lambda}\right) + (Ai * sN * i) == (1 - sN);$$

$$GPSO = \left(\frac{\mu}{i+\mu}\right)\left(\frac{c\,(1+\eta S)}{wLN} + A\mu * sS * \eta S * i\right) == (1 - sS) * \eta S;$$

```
"COMPARATIVE STATICS";
```

```
"I express these as functions to conduct comparative statics";
```

F1[i_, μ_, dr_, ηS_, λ_, σi_, σμ_, Ai_, Aμ_, sS_, sN_] :=

$$Ai\,i\,sN + \frac{i\,(Ai\,sN\,(dr + 2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu)}{(-1+\lambda)\,(i+\mu)} - (1 - sN);$$

F2[i_, μ_, dr_, ηS_, λ_, σi_, σμ_, Ai_, Aμ_, sS_, sN_] :=

$$\frac{\mu\left(A\mu\,i\,sS\,\eta S + \frac{Ai\,sN\,\lambda\,(dr+2\,(i+\mu))\,\sigma i + A\mu\,(dr+i)\,sS\,\eta S\,\sigma\mu}{-1+\lambda}\right)}{i+\mu} - (1 - sS)\,\eta S;$$

```
"Let J define the gradient matrix with respect to i and μ";
```

```
J = {gradf[F1[i, μ, dr, ηS, λ, σi, σμ, Ai, Aμ, sS, sN] , {i, μ}],
  gradf[F2[i, μ, dr, ηS, λ, σi, σμ, Ai, Aμ, sS, sN] , {i, μ}]}; FullSimplify[J] // MatrixForm
```

$$\begin{pmatrix} \frac{\mathrm{Ai}\,\mathrm{sN}\,((-1+\lambda)\,(i+\mu)^2+(\mathrm{dr}\,\mu+2\,(i+\mu)^2)\,\sigma i)+\mathrm{A}\mu\,\mathrm{sS}\,\eta\mathrm{S}\,(i^2+\mathrm{dr}\,\mu+2\,i\,\mu)\,\sigma\mu}{(-1+\lambda)\,(i+\mu)^2} & -\frac{i\,(\mathrm{Ai}\,\mathrm{dr}\,\mathrm{sN}\,\sigma i+\mathrm{A}\mu\,(\mathrm{dr}+i)\,\mathrm{sS}\,\eta\mathrm{S}\,\sigma\mu)}{(-1+\lambda)\,(i+\mu)^2} \\ \frac{\mu\,(-\mathrm{Ai}\,\mathrm{dr}\,\mathrm{sN}\,\lambda\,\sigma i+\mathrm{A}\mu\,\mathrm{sS}\,\eta\mathrm{S}\,(-\mathrm{dr}\,\sigma\mu+\mu\,(-1+\lambda+\sigma\mu)))}{(-1+\lambda)\,(i+\mu)^2} & \frac{\mathrm{Ai}\,\mathrm{sN}\,\lambda\,(\mathrm{dr}\,i+2\,(i+\mu)^2)\,\sigma i+\mathrm{A}\mu\,i\,\mathrm{sS}\,\eta\mathrm{S}\,(\mathrm{dr}\,\sigma\mu+i\,(-1+\lambda+\sigma\mu))}{(-1+\lambda)\,(i+\mu)^2} \end{pmatrix}$$

"The above implies that GPNO is upward sloping and GPSO is downward sloping if and only if $-\mathrm{Ai}*\mathrm{dr}*\mathrm{sN}*\lambda*\sigma i + \mathrm{A}\mu*\mathrm{sS}*\eta\mathrm{S}(-\mathrm{dr}*\sigma\mu + \mu*(-1+\lambda+\sigma\mu))<0$. This requires $\mu>\frac{\mathrm{Ai}\,\mathrm{dr}\,\mathrm{sN}\,\lambda\,\sigma i + \mathrm{A}\mu\,\mathrm{dr}\,\mathrm{sS}\,\eta\mathrm{S}\,\sigma\mu}{\mathrm{A}\mu\,\mathrm{sS}\,\eta\mathrm{S}\,(-1+\lambda+\sigma\mu)}$. A sufficient condition is $\rho - n \to 0$";

"Let B define the gradient matrix with respect to Aμ";

```
B = {gradf[F1[i, μ, dr, ηS, λ, σi, σμ, Ai, Aμ, sS, sN] , {Aμ}],
  gradf[F2[i, μ, dr, ηS, λ, σi, σμ, Ai, Aμ, sS, sN] , {Aμ}]}; FullSimplify[B] // MatrixForm
```

$$\begin{pmatrix} \frac{i\,(\mathrm{dr}+i)\,\mathrm{sS}\,\eta\mathrm{S}\,\sigma\mu}{(-1+\lambda)\,(i+\mu)} \\ \frac{\mathrm{sS}\,\eta\mathrm{S}\,\mu\,\left(i+\frac{(\mathrm{dr}+i)\,\sigma\mu}{-1+\lambda}\right)}{i+\mu} \end{pmatrix}$$

"Using Cramer's rule, it follows that the impact of Aμ on i and μ can be found by the following matrix equation. To keep the expressions tidy, I set the subsidy rates to zero. This is an inconsequential simplification.";

```
σi = 1; σμ = 1;
```

```
impactAμ = -Inverse[J].B.{ΔAμ}; MatrixForm[impactAμ] // FullSimplify
```

$$\begin{pmatrix} -\frac{i\,\mathrm{sS}\,\Delta\mathrm{A}\mu\,\eta\mathrm{S}\,(\mathrm{A}\mu\,(\mathrm{dr}+i)\,\mathrm{sS}\,\eta\mathrm{S}\,(\mathrm{dr}+i\,\lambda)\,(i+\mu)+\mathrm{Ai}\,\mathrm{sN}\,(2\,i\,\lambda\,(i+\mu)^2+\mathrm{dr}^2\,(i\,\lambda+\mu)+\mathrm{dr}\,\lambda\,(i+\mu)\,(3\,i+2\,\mu)))}{\mathrm{Ai}\,\mathrm{A}\mu\,\mathrm{sN}\,\mathrm{sS}\,\eta\mathrm{S}\,(i+\mu)\,(i\,(\mathrm{dr}+2\,\mathrm{dr}\,\lambda+i\,\lambda\,(3+\lambda))+2\,(\mathrm{dr}+2\,i)\,\lambda\mu)+\mathrm{Ai}^2\,\mathrm{sN}^2\,\lambda\,(i+\mu)\,(2\,(1+\lambda)\,(i+\mu)^2+\mathrm{dr}\,(i+i\,\lambda+2\,\mu))+\mathrm{A}\mu^2\,i\,\mathrm{sS}^2\,\eta\mathrm{S}^2\,(i\,\lambda\,(i+\mu)+\mathrm{dr}\,(i+} \\ -\frac{\mathrm{sS}\,\Delta\mathrm{A}\mu\,\eta\mathrm{S}\,\mu\,(\mathrm{A}\mu\,\mathrm{sS}\,\eta\mathrm{S}\,(\mathrm{dr}^2+2\,\mathrm{dr}\,i+i^2\,\lambda)\,(i+\mu)+\mathrm{Ai}\,\mathrm{sN}\,(i\,\lambda\,(1+\lambda)\,(i+\mu)^2+\mathrm{dr}^2\,(i\,\lambda+\mu)+\mathrm{dr}\,(i+\mu)\,(i+2\,i\,\lambda+\mu+\lambda\,\mu)))}{\mathrm{Ai}\,\mathrm{A}\mu\,\mathrm{sN}\,\mathrm{sS}\,\eta\mathrm{S}\,(i+\mu)\,(i\,(\mathrm{dr}+2\,\mathrm{dr}\,\lambda+i\,\lambda\,(3+\lambda))+2\,(\mathrm{dr}+2\,i)\,\lambda\mu)+\mathrm{Ai}^2\,\mathrm{sN}^2\,\lambda\,(i+\mu)\,(2\,(1+\lambda)\,(i+\mu)^2+\mathrm{dr}\,(i+i\,\lambda+2\,\mu))+\mathrm{A}\mu^2\,i\,\mathrm{sS}^2\,\eta\mathrm{S}^2\,(i\,\lambda\,(i+\mu)+\mathrm{dr}\,(i+} \end{pmatrix}$$

```
{di, dμ} = %;
```

"Obviously, di/dAμ < 0, and dμ/dAμ < 0";

"I can now totally differentiate c, wLN and nN, and substitute for di and dμ.";

```
dc = D[c, i] di + D[c, μ] dμ + D[c, Aμ] ΔAμ; dwLN = D[wLN, i] di + D[wLN, μ] dμ + D[wLN, Aμ] ΔAμ;
dnN = D[nN, i] di + D[nN, μ] dμ + D[nN, Aμ] ΔAμ;
```

```
FullSimplify[{dc, dwLN, dnN}] // MatrixForm
```

$$\begin{pmatrix} \frac{sS\,\triangle A\mu\,\eta S\,\lambda\,(A\mu^2\,dr\,i\,(dr+i)\,sS^2\,\eta S^2\,\mu+Ai^2\,sN^2\,(i+\mu)\,(i\,(dr+i)\,(dr+2\,i)\,\lambda+2\,(dr\,(dr+i)+i\,(2\,dr+i)\,\lambda)\,\mu+2\,dr\,(1+\lambda)\,\mu^2)+Ai\,A\mu\,sN\,sS\,\eta S\,(i^3\,\lambda\,(i+\mu)}{(1+\eta S)\,(Ai\,A\mu\,sN\,sS\,\eta S\,(i+\mu)\,(i\,(dr+2\,dr\,\lambda+i\,\lambda\,(3+\lambda))+2\,(dr+2\,i)\,\lambda\,\mu)+Ai^2\,sN^2\,\lambda\,(i+\mu)\,(2\,(1+\lambda)\,(i+\mu)^2+dr\,(i+i\,\lambda+2\,\mu))+A\mu^2\,i\,sS} \\ \frac{Ai\,sN\,sS\,\triangle A\mu\,\eta S\,(-1+\lambda)\,\lambda\,(Ai^2\,(dr+i)\,sN^2\,\lambda\,(i+\mu)\,(dr+2\,(i+\mu))\,(2\,(1+\lambda)\,(i+\mu)^2+dr\,(i+i\,\lambda+2\mu))+A\mu^2\,(dr+i)\,sS^2\,\eta S^2\,(2\,i^2\,\lambda\,(i+\mu)^2+2\,dr\,i\,(1+\lambda)\,(}{(A\mu\,(dr+i)\,sS\,\eta S+Ai\,sN\,\lambda\,(dr+2\,(i+\mu)))^2\,(Ai\,A\mu\,sN} \\ \frac{i\,sS\,\triangle A\mu\,\eta S\,(-1+\lambda)\,\mu\,(-A\mu\,dr\,i\,sS\,\eta S-Ai\,sN\,(dr-i\,\lambda)\,(i+\mu))}{(i+\mu)\,(Ai\,A\mu\,sN\,sS\,\eta S\,(i+\mu)\,(i\,(dr+2\,dr\,\lambda+i\,\lambda\,(3+\lambda))+2\,(dr+2\,i)\,\lambda\,\mu)+Ai^2\,sN^2\,\lambda\,(i+\mu)\,(2\,(1+\lambda)\,(i+\mu)^2+dr\,(i+i\,\lambda+2\,\mu))+A\mu^2\,i\,sS^2\,\eta S^2\,(i\,\lambda\,(i+\mu)+dr} \end{pmatrix}$$

```
"Obviously, dc/dAμ > 0, and dwLN/dAμ > 0, whereas dnN/dAμ > O
iff -Aμ*dr*i*sS*ηS - Ai*sN*
   (dr-i*λ)*(i+μ) > 0.A sufficient condition is ρ-n → 0.";
```

```
"APPENDIX E: MATHEMATICA PROGRAM FOR
   THE MODEL WITH FDI: THE CASE OF NO FRAGMENTATION β =1";
```

```
"- This program requires Mathematica Version 5.0.";

"MODEL: EXTENSION A
- Northern entrepreneurs can engage in FDI to shift production to the South.
- There is no fragmentation, β is set to 1 below.
- For innovation deterring, both
   Northern firms and MNFs use Northern specialized labor.
- For imitation-deterring, both Northern firms and MNFs use Southern specialized labor.
- Notes: For convenience, the subscripts and superscripts of the main model are entered
   in regular format. The parameter 'ng' stands for the rate of population growth. σi,
   σμ,σϕ stand for the rate of innovation, imitation and FDI subsidy, as in the paper.
- The expressions highlighted in red show the modifications
   and additions with the FDI model.
- Objective: To conduct comparative steady-state analysis for an increase in Aμ";
```

```
"THE BENCHMARK PARAMETERS";
```

```
"First, I clear the parameters and variables";

πN =.; πF =.; πSF =.; πSN =.
i =.; μF =.; μN =.; ϕ =.; m =.; f =.; c =.;
vN =.; vF =.; vSF =.; vSN =.;
XiN =.; XμN =.; XiF =.; XμF =.; Xi =.;
nF =.; nN =.; nSF =.; nSN =.;
wLN =.; wHS =.; wHN =.;
ρ =.; λ =.; ng =.; α =.; β =.; χ =.;
sS =.; sN =.;
ai =.; aϕ =.; aμF =.; aμN =.; γi =.; γμN =.; γμF =.; δi =.; δμN =.; δμF =.;
NN =.; NS =.; σi =.; σμ =.; σϕ =.;

"Benchmark parameters are as follows";

ρ = 0.07; λ = 1.25; ng = 0.014; α = 1.05; β = 1.0;
sS = 0.02; sN = 0.04;
ai = 0.9; aμN = 16; aμF = 8; aϕ = 0.6;
γi = 1; γμN = 8; γμF = 8;
δi = 1; δμN = 1; δμF = 2;
NN = 1; NS = 2; σi = 0; σμ = 0; σϕ = 0;
```

```
"THE MAIN EQUATIONS OF THE MODEL";
```

"The industry fractions are ";

$nN = \frac{i}{(i + \phi + \mu N)}$; $nF = \frac{\phi * i}{(i + \mu F) * (i + \phi + \mu N)}$; $nSF = \frac{\mu F * \phi}{(i + \phi + \mu N)\ (i + \mu F)}$; $nSN = \frac{\mu N}{(i + \phi + \mu N)}$;

"The aggregate imitation and FDI intensities are";

$m = (\mu N * nN) + (\mu F * nF)$; $F = \phi * nN$;

"Equilibrium in the Northern
specialized market requires sN*NN=(γi*Xi), which implies";

$Xi = (sN * NN) / \gamma i$;

"By construction, innovation-deterring in
the North implies Xi = (nN*XiN)+(nF*XiF), which gives";

$XiF = (Xi - (XiN * nN)) / nF$;

"Equilibrium in the Southern specialized market
requires sS*NS = nN*XμN*γμN + nF*XμF*γμF, which implies";

$X\mu F = \frac{(sS * NS) - (nN * X\mu N * \gamma\mu N)}{nF * \gamma\mu F}$;

"In the above system all difficulty levels (Xs) are expressed in terms of XiN and XμN";

"The profit flow of a Northern producer is ";

$\pi N = \left(\frac{(\lambda - wLN) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiN) - (wHS * \gamma\mu N * X\mu N)$;

"The marginal cost of production for a Multinational is"; $MCF = (1 - \beta)\ wLN + (\alpha * \beta)$;

"The profit flow of a Multinational producer is";

$\pi F = \left(\frac{(\lambda - MCF) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiF) - (wHS * \gamma\mu F * X\mu F)$;

"The profit flow of a Southern imitator succeeding a Northern producer is";

$\pi SN = \left(\frac{(wLN - 1) * c * (NN + NS)}{wLN}\right)$;

"The profit flow of a Southern imitator succeeding a Multinational producer is";

$\pi SF = \left(\frac{(MCF - 1) * c * (NN + NS)}{MCF}\right)$;

"With Di = δi*Xi/ng, the zero-profit condition in innovation implies";

$inn = \frac{vN * ng}{\delta i * Xi} == wLN * ai * (1 - \sigma i)$;

"From this I obtain an expression for vN"; Solve[{inn}, vN];

$$vN = -\frac{ai\ NN\ sN\ wLN\ \delta i\ (-1 + \sigma i)}{ng\ \gamma i};$$

"With DμN = δμN*XμN*nN/ng, the zero-profit
condition in imitation targeting a Northern firm implies";

$$imitN = \frac{vSN * ng}{\delta\mu N * X\mu N * nN} == a\mu N * (1 - \sigma\mu);$$

"From this I obtain an expression for vSN"; Solve[{imitN}, vSN];

$$vSN = -\frac{a\mu N\ i\ X\mu N\ \delta\mu N\ (-1 + \sigma\mu)}{ng\ (i + \mu N + \phi)};$$

"DμF = δμF*XμF*nF/ng, the zero-profit
condition in imitation targeting Multinational firm implies";

$$imitF = \frac{vSF * ng}{\delta\mu F * X\mu F * nF} == a\mu F * (1 - \sigma\mu);$$

"From this I obtain an expression for vSF"; Solve[{imitF}, vSF];

$$vSF = -\frac{a\mu F\ \delta\mu F\ (-1 + \sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)}{ng\ \gamma\mu F\ (i + \mu N + \phi)};$$

"Optimal level of FDI requires"; $vF = vN + \left(a\phi * (1 - \sigma\phi)\ \frac{c * (NN + NS)}{\lambda}\right);$

"Stock market valuations for each firm type (Northern, Multinational,
Southern succeeding Northern, and Southern succeeding Multinational)
combined with the relevant zero-profit conditions imply respectively";

$$vaN = vN == \frac{\pi N + \left(\phi * \left(vF - vN - \left(a\phi * (1 - \sigma\phi)\ \frac{c * (NN + NS)}{\lambda}\right)\right)\right)}{((\rho - ng) + i + \mu N)};$$

$$vaF = vF == \frac{\pi F}{((\rho - ng) + i + \mu F)};$$

$$vaSN = vSN == \frac{\pi SN}{((\rho - ng) + i)};$$

$$vaSF = vSF == \frac{\pi SF}{((\rho - ng) + i)};$$

"From the vaSF equation, I obtain an expression for c"; Solve[vaSF, c];

$$c = -\frac{a\mu F\ \alpha\ (-wLN - \beta + wLN\ \beta)\ \delta\mu F\ (i - ng + \rho)\ (-1 + \sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)}{ng\ (NN + NS)\ (1 - wLN\ \alpha - \alpha\ \beta + wLN\ \alpha\ \beta)\ \gamma\mu F\ (i + \mu N + \phi)};$$

"Foc for optimal innovation-deterring by the Northern incumbent is";

$$rpaINN = \frac{vN}{Xi} == \frac{wHN * \gamma i}{i * nN};$$

"From this I obtain an expression for wHN"; Solve[rpaINN, wHN];

$$wHN = -\frac{ai\, i^2\, wLN\, \delta i\, (-1 + \sigma i)}{ng\, \gamma i\, (i + \mu N + \phi)};$$

"Foc for optimal imitation-deterring by Northern incumbent is";

$$rpaIMN = \frac{vN}{X\mu N} == \frac{wHS * \gamma\mu N}{\mu N};$$

"From this I obtain an expression for wHS"; Solve[rpaIMN, wHS];

$$wHS = -\frac{ai\, NN\, sN\, wLN\, \delta i\, \mu N\, (-1 + \sigma i)}{ng\, X\mu N\, \gamma i\, \gamma\mu N};$$

"Foc for optimal innovation-deterring by Multinationals is";

$$rpaINF = \frac{vF}{Xi} == \frac{wHN * \gamma i}{i * nF};$$

"Foc for optimal imitation-deterring by Multinationals is";

$$rpaIMF = \frac{vF}{X\mu F} == \frac{wHS * \gamma\mu F}{\mu F};$$

"The labor market equilibrium for general purpose labor in the North is";

$$genNL = \left(\frac{nN * c * (NN + NS)}{\lambda}\right) + \left(\frac{(1 - \beta) * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{i * ai * \delta i * Xi}{ng}\right) == (1 - sN)\, NN;$$

"The labor market equilibrium for general purpose labor in the South is";

$$genSL = \left(\frac{nSN * c * (NN + NS)}{wLN}\right) + \left(\frac{nSF * c * (NN + NS)}{MCF}\right) + \left(\frac{\beta * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{\mu N * nN * a\mu N * \delta\mu N * X\mu N * nN}{ng}\right) + \left(\frac{\mu F * nF * a\mu F * \delta\mu F * X\mu F * nF}{ng}\right) + \left(\phi * nN * a\phi * \frac{c * (NN + NS)}{\lambda}\right) == (1 - sS)\, NS;$$

"Below are the expressions for aggregate MNF activity, and rent protection expenditures for Northern and Multinational firm respectively";

$\chi EQ = \chi == \beta * nF$; $RPAN = (wHN * \gamma i * XiN) + (wHS * \gamma\mu N * X\mu N)$;

$RPAF = (wHN * \gamma i * XiF) + (wHS * \gamma\mu F * X\mu F)$;

"The numerical solution to the model can be generated by ";

```
FindRoot[{vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, χEQ},
 {i, 0.06}, {μF, 0.03}, {μN, 0.03}, {ϕ, 0.1}, {XμN, 0.007},
 {XiN, 0.007}, {wLN, 1.15}, {χ, 0.34}, MaxIterations → 100000]

{i → 0.0244895, μF → 0.0123348, μN → 0.0238302, ϕ → 0.0236301,
 XμN → 0.00967962, XiN → 0.0490178, wLN → 1.10132, χ → 0.218414}

Cikti[1] = %;

{"nN" → nN, "nF" → nF, "nSN" → nSN, "nSF" → nSF, "F" → F, "m" → m,
  "wLN" → wLN, "c" → c, "RPAN" → RPAN, "RPAF" → RPAF, χ "→χ,vN" → vN, "vF" → vF,
  "vSF" → vSF, "vSN" → vSN, "πN" → πN, "πF" → πF, "πSF" → πSF, "πSN" → πSN,
  "wHS" → wHS, "wHN" → wHN, "XiN" → XiN, "XμN" → XμN, "XiF" → XiF, "XμF" → XμF} /. %

{nN → 0.340369, nF → 0.218414, nSN → 0.331206, nSF → 0.110011, F → 0.00804297,
 m → 0.0108052, wLN → 1.10132, c → 1.0981, RPAN → 0.0964139, RPAF → 0.117435,
 0.218414 →χ,vN → 2.83196, vF → 4.41323, vSF → 1.94898, vSN → 3.76531, πN → 0.29543,
 πF → 0.409656, πSF → 0.156872, πSN → 0.303068, wHS → 0.871499, wHN → 0.590144,
 XiN → 0.0490178, XμN → 0.00967962, XiF → 0.106751, XμF → 0.00780789}

"The line below quickly checks for all endogenous variables being positive";
```

```
y = {i, μF, μN, ϕ, vN, vF, vSF, vSN, nN, nF, nSF, nSN,
   m, F, χ, wHS, wHN, wLN, XiF, XμF, XμN, XiN} /. Cikti[1]; Positive[y]

{True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True, True}

"EFFECTS OF STRENGTHENING IPRS PROTECTION:
 A 10% increase in both aμN and aμF ";

πN =.; πF =.; πSF =.; πSN =.; i =.; μF =.; μN =.; ϕ =.; m =.; f =.; c =.; vN =.;
vF =.; vSF =.; vSN =.; XiN =.; XμN =.; XiF =.; XμF =.; Xi =.; nF =.; nN =.;
nSF =.; nSN =.; wLN =.; wHS =.; wHN =.; ρ =.; λ =.; ng =.; α =.; sS =.; sN =.;
ai =.; aϕ =.; aμF =.; aμN =.; σi =.; σμ =.; γi =.;
γμN =.; γμF =.; δi =.; δμN =.; δμF =.; NN =.; NS =.; χ =.;

ρ = 0.07; λ = 1.25; ng = 0.014; α = 1.05; β = 1.0;
 sS = 0.02; sN = 0.04;
ai = 0.9; aμN = 16 * 1.1; aμF = 8 * 1.1; aϕ = 0.6;
γi = 1; γμN = 8; γμF = 8;
δi = 1; δμN = 1; δμF = 2;
NN = 1; NS = 2; σi = 0; σμ = 0; σϕ = 0;
```

$$nN = \frac{i}{(i+\phi+\mu N)};\ nF = \frac{\phi * i}{(i+\mu F)*(i+\phi+\mu N)};\ nSF = \frac{\mu F*\phi}{(i+\phi+\mu N)\ (i+\mu F)};$$

$$nSN = \frac{\mu N}{(i+\phi+\mu N)};\ m = (\mu N * nN) + (\mu F * nF);\ F = \phi * nN;\ Xi = (sN * NN)/\gamma i;$$

$$XiF = (Xi - (XiN * nN))/nF;\ X\mu F = \frac{(sS * NS) - (nN * X\mu N * \gamma\mu N)}{nF * \gamma\mu F};$$

$$\pi N = \left(\frac{(\lambda - wLN) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiN) - (wHS * \gamma\mu N * X\mu N);\ MCF = \alpha\ ((1-\beta)\ wLN + \beta);$$

$$\pi F = \left(\frac{(\lambda - MCF) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiF) - (wHS * \gamma\mu F * X\mu F);$$

$$\pi SN = \left(\frac{(wLN - 1) * c * (NN + NS)}{wLN}\right);\ \pi SF = \left(\frac{(MCF - 1) * c * (NN + NS)}{MCF}\right);$$

$$vN = -\frac{ai\ NN\ sN\ wLN\ \delta i\ (-1+\sigma i)}{ng\ \gamma i};\ vSN = -\frac{a\mu N\ i\ X\mu N\ \delta\mu N\ (-1+\sigma\mu)}{ng\ (i+\mu N+\phi)};$$

$$vSF = -\frac{a\mu F\ \delta\mu F\ (-1+\sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)}{ng\ \gamma\mu F\ (i+\mu N+\phi)};$$

$$vF = vN + \left(a\phi * (1-\sigma\phi)\ \frac{c * (NN + NS)}{\lambda}\right);\ vaN = vN == \frac{\pi N + \left(\phi * \left(vF - vN - \left(a\phi * (1-\sigma\phi)\ \frac{c*(NN+NS)}{\lambda}\right)\right)\right)}{((\rho - ng) + i + \mu N)};$$

$$vaF = vF == \frac{\pi F}{((\rho - ng) + i + \mu F)};\ vaSN = vSN == \frac{\pi SN}{((\rho - ng) + i)};$$

$$c = -\frac{a\mu F\ \alpha\ (-wLN - \beta + wLN\ \beta)\ \delta\mu F\ (i - ng + \rho)\ (-1+\sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)}{ng\ (NN + NS)\ (1 - wLN\ \alpha - \alpha\ \beta + wLN\ \alpha\ \beta)\ \gamma\mu F\ (i+\mu N+\phi)};$$

$$wHN = -\frac{ai\ i^2\ wLN\ \delta i\ (-1+\sigma i)}{ng\ \gamma i\ (i+\mu N+\phi)};\ wHS = -\frac{ai\ NN\ sN\ wLN\ \delta i\ \mu N\ (-1+\sigma i)}{ng\ X\mu N\ \gamma i\ \gamma\mu N};$$

$$rpaINF = \frac{vF}{Xi} == \frac{wHN * \gamma i}{i * nF};\ rpaIMF = \frac{vF}{X\mu F} == \frac{wHS * \gamma\mu F}{\mu F};$$

$$genNL = \left(\frac{nN * c * (NN + NS)}{\lambda}\right) + \left(\frac{(1-\beta) * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{i * ai * \delta i * Xi}{ng}\right) == (1 - sN)\ NN;$$

$$genSL = \left(\frac{nSN * c * (NN + NS)}{wLN}\right) + \left(\frac{nSF * c * (NN + NS)}{MCF}\right) + \left(\frac{\beta * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{\mu N * nN * a\mu N * \delta\mu N * X\mu N * nN}{ng}\right) + \left(\frac{\mu F * nF * a\mu F * \delta\mu F * X\mu F * nF}{ng}\right) + \left(\phi * nN * a\phi * \frac{c * (NN + NS)}{\lambda}\right) == (1 - sS)\ NS;$$

$$\chi EQ = \chi == \beta * nF;\ RPAN = (wHN * \gamma i * XiN) + (wHS * \gamma\mu N * X\mu N);$$

$$RPAF = (wHN * \gamma i * XiF) + (wHS * \gamma\mu F * X\mu F);$$

"Below NTOT= nN+nF, captures the industries commanded by North.
MANS= 1-nN-(1-β)*nF= nSF+nSN+nFβ, captures manufacturing located in the South";

```
FindRoot[{ vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, χEQ},
 {i, 0.06}, {μF, 0.03}, {μN, 0.03}, {ϕ, 0.1}, {XμN, 0.007},
 {XiN, 0.007}, {wLN, 1.15}, {χ, 0.34}, MaxIterations → 100000]

{i → 0.0226078, μF → 0.0105638, μN → 0.0225177, ϕ → 0.0213223,
 XμN → 0.010003, XiN → 0.0170191, wLN → 1.11297, χ → 0.218699}

dχ = (χ /. %) - (χ /. Cikti[1]);  di = (i /. %) - (i /. Cikti[1]); dμF = (μF /. %) - (μF /. Cikti[1]);
dμN = (μN /. %) - (μN /. Cikti[1]); dϕ = (ϕ /. %) - (ϕ /. Cikti[1]);
dm = (m /. %) - (m /. Cikti[1]); dF = (F /. %) - (F /. Cikti[1]); dnN = (nN /. %) - (nN /. Cikti[1]);
dnN = (nN /. %) - (nN /. Cikti[1]); dNTOT = (((nN + nF) /. %) - (nN + nF) /. Cikti[1]);
dMANS = ((1 - nN - ((1 - β) * nF)) /. %) - ((1 - nN - ((1 - β) * nF)) /. Cikti[1]);
dnSN = (nSN /. %) - (nSN /. Cikti[1]); dnSF = (nSF /. %) - (nSF /. Cikti[1]);
dnF = (nF /. %) - (nF /. Cikti[1]); dwLN = (wLN /. %) - (wLN /. Cikti[1]);

y = {i, μF, μN, ϕ, vN, vF, vSF, vSN, nN, nF, nSF,
   nSN, m, F, wHS, wHN, wLN, XiF, XμF, XμN, XiN} /. %%; Positive[y]

{True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True}

{"nN" → nN, "nF" → nF, "nSN" → nSN, "nSF" → nSF, "F" → F, "m" → m, "wLN" → wLN,
  "c" → c, "RPAN" → RPAN, "RPAF" → RPAF, "χ" → χ, "vN" → vN, "vF" → vF, "vSF" → vSF,
  "vSN" → vSN, "πN" → πN, "πF" → πF, "πSF" → πSF, "πSN" → πSN, "wHS" → wHS,
  "wHN" → wHN, "XiN" → XiN, "XμN" → XμN, "XiF" → XiF, "XμF" → XμF} /. %%%

{nN → 0.340234, nF → 0.218699, nSN → 0.338878, nSF → 0.10219, F → 0.00725458,
 m → 0.00997157, wLN → 1.11297, c → 1.10447, RPAN → 0.0738104, RPAF → 0.13312,
 χ → 0.218699, vN → 2.86193, vF → 4.45236, vSF → 2.00719, vSN → 4.27852,
 πN → 0.289414, πF → 0.397024, πSF → 0.157781, πSN → 0.336325, wHS → 0.805306,
 wHN → 0.550345, XiN → 0.0170191, XμN → 0.010003, XiF → 0.156423, XμF → 0.0073006}

{{"dχ" → dχ}, {"di" → di}, {"dμF" → dμF}, {"dμN" → dμN}, {"dϕ" → dϕ},
 {"dm" → dm}, {"dF" → dF}, {"dnN" → dnN}, {{"dNTOT" → dNTOT}}, {{"dMANS" → dMANS}},
 {"dnSN" → dnSN}, {"dnSF" → dnSF}, {"dnF" → dnF}, {"dwLN" → dwLN}}

{{dχ → 0.000284369}, {di → -0.00188173}, {dμF → -0.00177107},
 {dμN → -0.00131254}, {dϕ → -0.0023078}, {dm → -0.000833619}, {dF → -0.00078839},
 {dnN → -0.00013531}, {{dNTOT → 0.000149059}}, {{dMANS → 0.00013531}},
 {dnSN → 0.00767183}, {dnSF → -0.00782089}, {dnF → 0.000284369}, {dwLN → 0.0116533}}

"Observe that all qualitative changes conform with Proposition 1. Hence,
   one can add the following line and check the robustness of the results by
   re-running the simulations with high and low values for the parameters.";
```

```
If[dwLN > 0 && di < 0 && dμF < 0 && dμN < 0 && dm < 0 && dF < 0 && dNTOT > 0, OK, NO]
```

OK

```
"It should be noted that the uniqueness of the results in the FindRoot
   line above is checked by changing the initial levels. Extensive numerical
   simulations implied that the solution generated by the FindRoot command
   with all of the endogenous variables strictly positive is unique.";
```

```
"MATHEMATICA PROGRAM FOR THE MODEL WITH
   FDI: THE CASE OF ENDOGENOUS FRAGMENTATION, β ENDOGENOUS";
```

```
"MODEL: EXTENSION B:
- Northern entrepreneurs can engage in FDI to shift production to the South
- There is fragmentation of production. The scale of
   within Multinational fragmentation β is endogenously determined.
- aϕ is now endogenously determined as a function of β, with aϕ' > 0 and aϕ''>0.
- Both Northern firms and Multinationals can be targeted by the Southern imitators.
- For innovation deterring, both
   Northern firms and MNFs use Northern specialized labor.
- For imitation-deterring, both Northern firms and MNFs use Southern specialized labor.
- The expressions highlighted in red show the
   modifications and additions due to including FDI and endogenous β.
- Objective: To conduct comparative steady-state analysis for an increase in Aμ";
```

```
"THE BENCHMARK PARAMETERS";
```

```
"First, I clear the variables";
```

```
πN =.; πF =.; πSF =.; πSN =.; i =.; μF =.; μN =.; ϕ =.; m =.; f =.; c =.; vN =.;
vF =.; vSF =.; vSN =.; XiN =.; XμN =.; XiF =.; XμF =.; Xi =.; nF =.; nN =.;
nSF =.; nSN =.; wLN =.; wHS =.; wHN =.; ρ =.; λ =.; ng =.; α =.; sS =.; sN =.;
ai =.; aϕ =.; aμF =.; aμN =.; σi =.; σμ =.; γi =.; γμN =.; γμF =.;
δi =.; δμN =.; δμF =.; NN =.; NS =.; α =.; β =.; aϕ =.; ψ =.; Aϕ =.;
```

```
"Note: The benchmark parameters are the same as the exogenous fragmentation
   model. The only exception is that I now assume a functional form for aϕ.";
```

```
ρ = 0.07; λ = 1.25; ng = 0.014; α = 1.05; β =.;
 sS = 0.02; sN = 0.04;
ai = 0.9; aϕ =.; ψ = 3.0; Aϕ = 0.02;
γi = 1; γμN = 8; γμF = 8;
δi = 1; δμN = 1; δμF = 2;
NN = 1; NS = 2; σi = 0; σϕ = 0; σμ = 0;
```

```
aμN = 16 * 1.00; aμF = 8 * 1.00;
```

```
"I assume the following functional form for aϕ";
```

$$a\phi = A\phi * \left(\frac{\beta}{1-\beta}\right)^{\psi};\ a\phi 1 = D[a\phi,\ \beta];\ \text{"where } A\phi>0 \text{ and } \psi>1\text{"};$$

```
"This functional form satisfies  aϕ' > 0 and aϕ''>0 as seen below";
```

```
{aϕ1 = D[aϕ, β], D[aϕ1, β]} // FullSimplify
```

$$\left\{\frac{0.06\left(\frac{\beta}{1-\beta}\right)^{2.}}{(-1.+\beta)^{2}},\ \frac{-0.12\,\beta-0.12\,(-1.+\beta)^{2}\left(\frac{\beta}{1-\beta}\right)^{2.}}{(-1.+\beta)^{5}}\right\}$$

```
"The first order condition that gives the optimal level of β is";
```

$$F\beta = \frac{a\phi 1 * \beta}{a\phi}\,(1-\sigma\phi) == \frac{(wLN-\alpha)*\beta}{(\rho+i+\mu F-ng)*a\phi};\ \chi = \beta * nF;$$

```
"THE MAIN EQUATIONS OF THE MODEL (CONDENSED)";
```

$$nN = \frac{i}{(i+\phi+\mu N)};\; nF = \frac{\phi * i}{(i+\mu F) * (i+\phi+\mu N)};\; nSF = \frac{\mu F * \phi}{(i+\phi+\mu N)\,(i+\mu F)};$$

$$nSN = \frac{\mu N}{(i+\phi+\mu N)};\; m = (\mu N * nN) + (\mu F * nF);\; F = \phi * nN;\; Xi = (sN * NN)/\gamma i;$$

$$XiF = (Xi - (XiN * nN))/nF;\; X\mu F = \frac{(sS * NS) - (nN * X\mu N * \gamma\mu N)}{nF * \gamma\mu F};$$

$$\pi N = \left(\frac{(\lambda - wLN) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiN) - (wHS * \gamma\mu N * X\mu N);\; MCF = ((1-\beta)\, wLN + (\alpha * \beta));$$

$$\pi F = \left(\frac{(\lambda - MCF) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiF) - (wHS * \gamma\mu F * X\mu F);$$

$$\pi SN = \left(\frac{(wLN - 1) * c * (NN + NS)}{wLN}\right);\; \pi SF = \left(\frac{(MCF - 1) * c * (NN + NS)}{MCF}\right);$$

$$vN = -\frac{ai\, NN\, sN\, wLN\, \delta i\, (-1+\sigma i)}{ng\, \gamma i};\; vSN = -\frac{a\mu N\, i\, X\mu N\, \delta\mu N\, (-1+\sigma\mu)}{ng\, (i+\mu N+\phi)};$$

$$vSF = -\frac{a\mu F\, \delta\mu F\, (-1+\sigma\mu)\, (i\, NS\, sS - i\, X\mu N\, \gamma\mu N + NS\, sS\, \mu N + NS\, sS\, \phi)}{ng\, \gamma\mu F\, (i+\mu N+\phi)};$$

$$vF = vN + \left(a\phi * (1-\sigma\phi)\, \frac{c * (NN + NS)}{\lambda}\right);\; vaN = vN == \frac{\pi N + \left(\phi * \left(vF - vN - \left(a\phi * (1-\sigma\phi)\, \frac{c * (NN + NS)}{\lambda}\right)\right)\right)}{((\rho - ng) + i + \mu N)};$$

$$vaF = vF == \frac{\pi F}{((\rho - ng) + i + \mu F)};\; vaSN = vSN == \frac{\pi SN}{((\rho - ng) + i)};$$

$$c = -(a\mu F\, \alpha\, (-wLN - \beta + wLN\, \beta)\, \delta\mu F\, (i - ng + \rho)\, (-1+\sigma\mu)\, (i\, NS\, sS - i\, X\mu N\, \gamma\mu N + NS\, sS\, \mu N + NS\, sS\, \phi)) /$$
$$(ng\, (NN + NS)\, (1 - wLN\, \alpha - \alpha\, \beta + wLN\, \alpha\, \beta)\, \gamma\mu F\, (i + \mu N + \phi));\; wHN = -\frac{ai\, i^2\, wLN\, \delta i\, (-1+\sigma i)}{ng\, \gamma i\, (i+\mu N+\phi)};$$

$$wHS = -\frac{ai\, NN\, sN\, wLN\, \delta i\, \mu N\, (-1+\sigma i)}{ng\, X\mu N\, \gamma i\, \gamma\mu N};\; rpaINF = \frac{vF}{Xi} == \frac{wHN * \gamma i}{i * nF};\; rpaIMF = \frac{vF}{X\mu F} == \frac{wHS * \gamma\mu F}{\mu F};$$

$$genNL = \left(\frac{nN * c * (NN + NS)}{\lambda}\right) + \left(\frac{(1-\beta) * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{i * ai * \delta i * Xi}{ng}\right) == (1 - sN)\, NN;$$

$$genSL = \left(\frac{nSN * c * (NN + NS)}{wLN}\right) + \left(\frac{nSF * c * (NN + NS)}{MCF}\right) +$$
$$\left(\frac{\beta * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{\mu N * nN * a\mu N * \delta\mu N * X\mu N * nN}{ng}\right) +$$
$$\left(\frac{\mu F * nF * a\mu F * \delta\mu F * X\mu F * nF}{ng}\right) + \left(\phi * nN * a\phi * \frac{c * (NN + NS)}{\lambda}\right) == (1 - sS)\, NS;$$

$$RPAN = (wHN * \gamma i * XiN) + (wHS * \gamma\mu N * X\mu N);\; RPAF = (wHN * \gamma i * XiF) + (wHS * \gamma\mu F * X\mu F);$$

```
{ vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, Fβ};

FindRoot[{ vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, Fβ},
 {i, 0.02}, {μF, 0.03}, {μN, 0.02}, {ϕ, 0.07}, {XμN, 0.007},
 {XiN, 0.04}, {wLN, 1.10}, {β, 0.4}, MaxIterations → 100000]

{i → 0.0286495, μF → 0.0376048, μN → 0.0311775, ϕ → 0.0652849,
 XμN → 0.00989728, XiN → 0.102287, wLN → 1.07351, β → 0.48142}

Cikti[1] =%;
```

```
{"χ" → χ, "vN" → vN, "vF" → vF, "vSF" → vSF, "vSN" → vSN,
  "nN" → nN, "nF" → nF, "nSF" → nSF, "nSN" → nSN, "m" → m, "F" → F,
  "wHS" → wHS, "wHN" → wHN, "XiF" → XiF, "XμF" → XμF, "MANS" → MANS} /. %%

{χ → 0.108628, vN → 2.76045, vF → 2.80144, vSF → 3.12413, vSN → 2.59016, nN → 0.228991,
 nF → 0.22564, nSF → 0.296172, nSN → 0.249197, m → 0.0156245, F → 0.0149496,
 wHS → 1.08696, wHN → 0.452745, XiF → 0.0734674, XμF → 0.0121149, MANS → MANS}

{i, μN, μF, ϕ, nN, nF, nSN, nSF, F, m, wLN, c, RPAN, RPAF, β} /. %%%

{0.0286495, 0.0311775, 0.0376048, 0.0652849, 0.228991, 0.22564, 0.249197,
 0.296172, 0.0149496, 0.0156245, 1.07351, 1.06734, 0.132374, 0.13861, 0.48142}
```

```
y = {i, μF, μN, ϕ, vN, vF, vSF, vSN, nN, nF, nSF, nSN,
   m, F, wHS, wHN, wLN, XiF, XμF, XμN, XiN} /. Cikti[1]; Positive[y]
```

```
{True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True}
```

```
"EFFECTS OF STRENGTHENING IPRS PROTECTION:
 A 10% increase in both aμN and aμF ";
```

```
Clear { vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, Fβ}; πN =.; πF =.;
πSF =.; πSN =.; i =.; μF =.; μN =.; ϕ =.; m =.; f =.; c =.; vN =.; vF =.;
vSF =.; vSN =.; XiN =.; XμN =.; XiF =.; XμF =.; Xi =.; nF =.; nN =.; nSF =.;
nSN =.; wLN =.; wHS =.; wHN =.; ρ =.; λ =.; ng =.; α =.; sS =.; sN =.;
ai =.; aϕ =.; aμF =.; aμN =.; σi =.; σμ =.; γi =.; γμN =.; γμF =.;
δi =.; δμN =.; δμF =.; NN =.; NS =.; α =.; β =.; aϕ =.; ψ =.; Aϕ =.;
```

```
ρ = 0.07; λ = 1.25; ng = 0.014; α = 1.05; β =.;
 sS = 0.02; sN = 0.04;
ai = 0.9; aϕ =.; ψ = 3.0; Aϕ = 0.02;
γi = 1; γμN = 8; γμF = 8;
δi = 1; δμN = 1; δμF = 2;
NN = 1; NS = 2; σi = 0; σϕ = 0; σμ = 0;
```

```
aμN = 16 * 1.10; aμF = 8 * 1.10;
```

$$a\phi = A\phi * \left(\frac{\beta}{1-\beta}\right)^{\psi};\ a\phi 1 = D[a\phi,\ \beta];\ \chi = \beta * nF;$$

$$F\beta = \frac{a\phi 1 * \beta}{a\phi}\ (1 - \sigma\phi)\ == \frac{(wLN - \alpha) * \beta}{(\rho + i + \mu F - ng) * a\phi};$$

$$nN = \frac{i}{(i+\phi+\mu N)};\ nF = \frac{\phi * i}{(i+\mu F)*(i+\phi+\mu N)};\ nSF = \frac{\mu F * \phi}{(i+\phi+\mu N)\ (i+\mu F)};$$

$$nSN = \frac{\mu N}{(i+\phi+\mu N)};\ m = (\mu N * nN) + (\mu F * nF);\ F = \phi * nN;\ Xi = (sN * NN) / \gamma i;$$

$$XiF = (Xi - (XiN * nN)) / nF;\ X\mu F = \frac{(sS * NS) - (nN * X\mu N * \gamma\mu N)}{nF * \gamma\mu F};$$

$$\pi N = \left(\frac{(\lambda - wLN) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiN) - (wHS * \gamma\mu N * X\mu N);\ MCF = ((1-\beta)\ wLN + (\alpha * \beta));$$

$$\pi F = \left(\frac{(\lambda - MCF) * c * (NN + NS)}{\lambda}\right) - (wHN * \gamma i * XiF) - (wHS * \gamma\mu F * X\mu F);$$

$$\pi SN = \left(\frac{(wLN - 1) * c * (NN + NS)}{wLN}\right);\ \pi SF = \left(\frac{(MCF - 1) * c * (NN + NS)}{MCF}\right);$$

$$vN = -\frac{ai\ NN\ sN\ wLN\ \delta i\ (-1+\sigma i)}{ng\ \gamma i};\ vSN = -\frac{a\mu N\ i\ X\mu N\ \delta\mu N\ (-1+\sigma\mu)}{ng\ (i+\mu N+\phi)};$$

$$vSF = -\frac{a\mu F\ \delta\mu F\ (-1+\sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)}{ng\ \gamma\mu F\ (i+\mu N+\phi)};$$

$$vF = vN + \left(a\phi * (1-\sigma\phi)\ \frac{c * (NN + NS)}{\lambda}\right);\ vaN = vN == \frac{\pi N + \left(\phi * \left(vF - vN - \left(a\phi * (1-\sigma\phi)\ \frac{c * (NN + NS)}{\lambda}\right)\right)\right)}{((\rho - ng) + i + \mu N)};$$

$$vaF = vF == \frac{\pi F}{((\rho - ng) + i + \mu F)};\ vaSN = vSN == \frac{\pi SN}{((\rho - ng) + i)};$$

$$c = -(a\mu F\ \alpha\ (-wLN - \beta + wLN\ \beta)\ \delta\mu F\ (i - ng + \rho)\ (-1+\sigma\mu)\ (i\ NS\ sS - i\ X\mu N\ \gamma\mu N + NS\ sS\ \mu N + NS\ sS\ \phi)) /$$

$$(ng\ (NN + NS)\ (1 - wLN\ \alpha - \alpha\ \beta + wLN\ \alpha\ \beta)\ \gamma\mu F\ (i+\mu N+\phi));\ wHN = -\frac{ai\ i^2\ wLN\ \delta i\ (-1+\sigma i)}{ng\ \gamma i\ (i+\mu N+\phi)};$$

$$wHS = -\frac{ai\ NN\ sN\ wLN\ \delta i\ \mu N\ (-1+\sigma i)}{ng\ X\mu N\ \gamma i\ \gamma\mu N};\ rpaINF = \frac{vF}{Xi} == \frac{wHN * \gamma i}{i * nF};\ rpaIMF = \frac{vF}{X\mu F} == \frac{wHS * \gamma\mu F}{\mu F};$$

$$genNL = \left(\frac{nN * c * (NN + NS)}{\lambda}\right) + \left(\frac{(1-\beta) * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{i * ai * \delta i * Xi}{ng}\right) == (1 - sN)\ NN;$$

$$genSL = \left(\frac{nSN * c * (NN + NS)}{wLN}\right) + \left(\frac{nSF * c * (NN + NS)}{MCF}\right) + \left(\frac{\beta * nF * c * (NN + NS)}{\lambda}\right) + \left(\frac{\mu N * nN * a\mu N * \delta\mu N * X\mu N * nN}{ng}\right) + \left(\frac{\mu F * nF * a\mu F * \delta\mu F * X\mu F * nF}{ng}\right) + \left(\phi * nN * a\phi * \frac{c * (NN + NS)}{\lambda}\right) == (1 - sS)\ NS;$$

$$RPAN = (wHN * \gamma i * XiN) + (wHS * \gamma\mu N * X\mu N);\ RPAF = (wHN * \gamma i * XiF) + (wHS * \gamma\mu F * X\mu F);$$

"Below NTOT= nN+nF, captures the industries commanded by North.
MANS= 1-nN-(1-β)*nF= nSF+nSN+nFβ, captures manufacturing located in the South";

```
FindRoot[{vaN, vaF, vaSN, rpaINF, rpaIMF, genNL, genSL, Fβ},
 {i, 0.02}, {μF, 0.03}, {μN, 0.02}, {ϕ, 0.07}, {XμN, 0.007},
 {XiN, 0.04}, {wLN, 1.10}, {β, 0.4}, MaxIterations → 100000]

{i → 0.0272019, μF → 0.0328959, μN → 0.0300667, ϕ → 0.0587723,
 XμN → 0.0101856, XiN → 0.0638506, wLN → 1.08401, β → 0.516517}

dβ = (β /. %) - (β /. Cikti[1]); dχ = (χ /. %) - (χ /. Cikti[1]); di = (i /. %) - (i /. Cikti[1]);
dμF = (μF /. %) - (μF /. Cikti[1]); dμN = (μN /. %) - (μN /. Cikti[1]);
dϕ = (ϕ /. %) - (ϕ /. Cikti[1]); dm = (m /. %) - (m /. Cikti[1]); dF = (F /. %) - (F /. Cikti[1]);
dnN = (nN /. %) - (nN /. Cikti[1]); dnN = (nN /. %) - (nN /. Cikti[1]);
dNTOT = ((nN + nF) /. %) - ((nN + nF) /. Cikti[1]); dnSN = (nSN /. %) - (nSN /. Cikti[1]);
dnSF = (nSF /. %) - (nSF /. Cikti[1]); dnF = (nF /. %) - (nF /. Cikti[1]);
dwLN = (wLN /. %) - (wLN /. Cikti[1]); dNTOT = (((nN + nF) /. %) - (nN + nF) /. Cikti[1]);
dMANS = ((1 - nN - ((1 - β) * nF)) /. %) - ((1 - nN - ((1 - β) * nF)) /. Cikti[1]);

{"χ" → χ, "nN" → nN, "nF" → nF, "nSN" → nSN, "nSF" → nSF, "F" → F, "m" → m,
  "wLN" → wLN, "c" → c, "RPAN" → RPAN, "RPAF" → RPAF, "vN" → vN, "vF" → vF, "vSF" → vSF,
  "vSN" → vSN, "πN" → πN, "πF" → πF, "πSF" → πSF, "πSN" → πSN, "wHS" → wHS,
  "wHN" → wHN, "XiN" → XiN, "XμN" → XμN, "XiF" → XiF, "XμF" → XμF, "MANS" → MANS} /. %%

{χ → 0.11841, nN → 0.234417, nF → 0.229246, nSN → 0.259104, nSF → 0.277233, F → 0.0137772,
 m → 0.0145894, wLN → 1.08401, c → 1.07415, RPAN → 0.112182, RPAF → 0.142286,
 vN → 2.78746, vF → 2.85032, vSF → 3.28408, vSN → 3.00163, πN → 0.315732,
 πF → 0.330916, πSF → 0.200773, πSN → 0.249742, wHS → 1.02854, wHN → 0.444362,
 XiN → 0.0638506, XμN → 0.0101856, XiF → 0.109194, XμF → 0.0113953, MANS → MANS}

y = {i, μF, μN, ϕ, vN, vF, vSF, vSN, nN, nF, nSF,
   nSN, m, F, wHS, wHN, wLN, XiF, XμF, XμN, XiN} /. %%%; Positive[y]

{True, True, True, True, True, True, True, True, True, True,
 True, True, True, True, True, True, True, True, True, True, True}

{{"dβ" → dβ}, {"dχ" → dχ}, {"di" → di}, {"dμF" → dμF}, {"dμN" → dμN}, {"dϕ" → dϕ},
 {"dm" → dm}, {"dF" → dF}, {"dnN" → dnN}, {"dnSN" → dnSN}, {"dnSF" → dnSF},
 {"dnF" → dnF}, {"dwLN" → dwLN}, {{"dNTOT" → dNTOT}}, {{"dMANS" → dMANS}}}

{{dβ → 0.0350973}, {dχ → 0.00978193}, {di → -0.00144753}, {dμF → -0.00470894},
 {dμN → -0.00111075}, {dϕ → -0.00651258}, {dm → -0.00103512}, {dF → -0.00117244},
 {dnN → 0.00542579}, {dnSN → 0.00990753}, {dnSF → -0.0189393}, {dnF → 0.003606},
 {dwLN → 0.010504}, {{dNTOT → 0.00903179}}, {{dMANS → 0.000750134}}}
```

```
If[dwLN > 0 && di < 0 && dμF < 0 && dμN < 0 && dm  < 0 && dF <  0 && dNTOT > 0, OK, NO]
```

OK

```
"It should be noted that the uniqueness of the results in the FindRoot
   line above is checked by changing the initial levels. Extensive numerical
   simulations implied that the solution generated by the FindRoot command
   with all of the endogenous variables strictly positive is unique.";
```