Appendices [a]

for

Globalization, R&D and the iPod Cycle

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July 2008

Paper published in the *Journal of International Economics* 77 (2009) 101-108

[a] Not to considered for publication. To be made available on the authors' we sites and also upon request.

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In[106]:= "GLOBALIZATION, R&D and the iPod CYCLE";

In[107]:= "APPENDIX A:
ANALYTICAL DERIVATIONS OF COMPARATIVE STATICS
THE MODEL WITH EQUAL IMITATION
   RATES TARGETING BOTH NORTHERN AND OUTSOURCING INDUSTRIES;
Date: August 15, 2007";

In[108]:= "- This program requires Mathematica Version 5.0 or above. Before evaluating the
   cells, 'Math Econ' package written by Cliff Huang and Philip Crooke needs to
   be run. This package accompanies the book 'Mathematics and Mathematica for
   Economists', 1997, Blackwell Publishers: Oxford, written by the above authors.
- Notes: In this program, for convenience I enter the subscripts
   and superscripts of the main model in regular format.";

In[109]:= "We consider the model with Southern imitation targeting both the
   Northern and Outsourcing firms. I first write the Steady State equations
To get simpler terms I set ΣiO=(1-σiO) and ΣO=(1-σO) and also dr = ρ-n.
μO: imitation targeting Outsourcing firms
μN: imitation targeting Northern firms ";

In[110]:= Clear[FEIN, FEIO]; Clear[w, λ, aN, aO, kN, kO, c, λ, iN, iO, ΣiO, ΣO, mN, mO, dr, ηS, μi, μO];
Clear[LN, LS]; Clear[nN, nO, nS];

In[111]:= "I first solve for the steady-state industry shares";

In[112]:= "nN flows in/out"; EQ1 = iN (nO + nS) == nN (μN + iO);
"nO flows in/out"; EQ2 = iO (nN + nS) == nO (μO + iN);
"mass of one of industries"; EQ3 = nO + nN + nS == 1;

In[114]:= Solve[{EQ1, EQ2, EQ3}, {nO, nN, nS}] // FullSimplify
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Out[114]= $\left\{\left\{nO \to \frac{iO}{iN + iO + \mu O},\ nN \to \frac{iN}{iN + iO + \mu N},\ nS \to \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu O}\right\}\right\}$

In[115]:= $FEIN = w * \lambda * aN * kN == \frac{c\,(\lambda - (mN * w))}{(dr + iN + iO + \mu i)};$

In[116]:= $FEIO = w * \lambda * aO * \Sigma iO * kO == \frac{c\,(\lambda - (\Sigma O * mO))}{(dr + iN + iO + \mu O)};$

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In[117]:= "To capture the equal imitation rates, I set"; μi = μ; μO = μ;
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In[118]:= Solve[{FEIN, FEIO}, {w, c}] // FullSimplify

Out[118]= $\left\{\{c \to 0, w \to 0\}, \left\{c \to \frac{\lambda\,(dr + iN + iO + \mu)\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)}, w \to \frac{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,\Sigma iO}\right\}\right\}$

In[119]:= "The solution is the second vector. Note that w is in terms of parameters only. Hence the comparative statics from the no imitation model apply to w. The restrictions on parameters also remain intact as below";

In[120]:= "Note the following restrictions and the implications for the parameters.

1. For w>0, we need aO kO λ ΣiO + aN kN (-λ+mO ΣO)>0.

Put differently, aO kO λ ΣiO - aN kN (λ-mO ΣO)>0 ";

In[121]:= "2. For w>1, we need aO kO λ ΣiO + aN kN (-λ+mO ΣO)>aO kO mN ΣiO.

This can be simplified to aO kO ΣiO(λ-mN) > aN kN (λ-mO ΣO) ";

In[122]:= "3. The restriction that λ > mN w (required for positive Northern profits), implies w < λ/mN. This automatically holds and can be seen by eye inspection of the w expression.";

In[123]:= "4. The restriction that mN w > mO*Σo implies (λ-mO ΣO)(aO kO ΣiO - aNkN)>0.

This can be simplified to (aO kO ΣiO - aNkN) > 0;

This implies that the unit labor requirement in outsourcing-targeted R&D is larger that that in local-sourcing-targeted R&D. This makes intuitive sense. ";

In[124]:= "Now I set c and w equal to the solutions given above";

In[125]:= $c = \frac{\lambda\,(dr + iN + iO + \mu)\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)}$; $w = \frac{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,\Sigma iO}$;

In[126]:= "I again solve for the steady-state industry shares with μi=μO=μ imposed. We obtain";

In[127]:= "nN flows in/out"; EQ1 = iN (nO + nS) == nN (μi + iO);
"nO flows in/out"; EQ2 = iO (nN + nS) == nO (μO + iN);
"mass of one of industries"; EQ3 = nO + nN + nS == 1;

In[129]:= Solve[{EQ1, EQ2, EQ3}, {nO, nN, nS}] // FullSimplify

Out[129]= $\left\{\left\{nO \to \frac{iO}{iN + iO + \mu}, nN \to \frac{iN}{iN + iO + \mu}, nS \to \frac{\mu}{iN + iO + \mu}\right\}\right\}$

In[130]:= "I now set the industry shares to the solutions";
$nO = \frac{iO}{iN + iO + \mu}$; $nN = \frac{iN}{iN + iO + \mu}$; $nS = \frac{\mu}{iN + iO + \mu}$;

In[132]:= "I now rewrite the Northern and Southern labor market equations ";

In[133]:= $$LN = (iN * aN * kN) + (iO * aO * kO) + \left(nN * \frac{c * mN}{\lambda}\right) - \frac{1}{(1 + \eta S)} \;\text{// FullSimplify}$$

Out[133]= $$aO\, iO\, kO - \frac{1}{1 + \eta S} - \frac{aN\, dr\, iN\, kN}{iN + iO + \mu} + \frac{aO\, iN\, kO\, \lambda\, (dr + iN + iO + \mu)\, \Sigma iO}{(iN + iO + \mu)\, (\lambda - mO\, \Sigma O)}$$

In[134]:= $$LS = \frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)} \;\text{// FullSimplify}$$

Out[134]= $$-1 + \frac{1}{1 + \eta S} + \frac{(dr + iN + iO + \mu)\, (iO\, mO + \lambda\, \mu)\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}{mN\, (iN + iO + \mu)\, (\lambda - mO\, \Sigma O)}$$

In[135]:= **COMPARATIVE STATICS;**

In[136]:= F1[iN_, iO_, μ_, aO_, aN_, mN_, mO_, ηS_, λ_, dr_, kN_, kO_, ΣO_, ΣiO_] :=

$$-\frac{1}{1 + \eta S} - \frac{aN\, dr\, iN\, kN}{iN + iO + \mu} + aO\, kO \left(iO + \frac{iN\, \lambda\, (dr + iN + iO + \mu)\, \Sigma iO}{(iN + iO + \mu)\, (\lambda - mO\, \Sigma O)}\right);$$

In[137]:= F2[iN_, iO_, μ_, aO_, aN_, mN_, mO_, ηS_, λ_, dr_, kN_, kO_, ΣO_, ΣiO_] :=

$$-1 + \frac{1}{1 + \eta S} + \frac{(dr + iN + iO + \mu)\, (iO\, mO + \lambda\, \mu)\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}{mN\, (iN + iO + \mu)\, (\lambda - mO\, \Sigma O)};$$

In[138]:= Clear[J, B]

In[139]:= J = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {iN, iO}],
gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {iN, iO}]};

In[140]:= FullSimplify[J] // MatrixForm

Out[140]//MatrixForm=

$$\begin{pmatrix} \frac{aO\, kO\, \lambda \left(dr\, (iO+\mu) + (iN+iO+\mu)^2\right) \Sigma iO - aN\, dr\, kN\, (iO+\mu)\, (\lambda - mO\, \Sigma O)}{(iN+iO+\mu)^2\, (\lambda - mO\, \Sigma O)} & \frac{aN\, dr\, iN\, kN}{(iN+iO+\mu)^2} + aO \left(kO + \frac{dr\, iN\, kO\, \lambda\, \Sigma iO}{(iN+iO+\mu)^2\, (-\lambda + mO\, \Sigma O)}\right) \\ -\frac{dr\, (iO\, mO + \lambda\, \mu)\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}{mN\, (iN+iO+\mu)^2\, (\lambda - mO\, \Sigma O)} & \frac{\left(mO\, (iN+iO+\mu)^2 + dr\, (iN\, mO + (mO - \lambda)\, \mu)\right) (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}{mN\, (iN+iO+\mu)^2\, (\lambda - mO\, \Sigma O)} \end{pmatrix}$$

In[141]:= "To see the slope clearly I set"; dr = 0; FullSimplify[J] // MatrixForm

Out[141]//MatrixForm=

$$\begin{pmatrix} \frac{aO\, kO\, \lambda\, \Sigma iO}{\lambda - mO\, \Sigma O} & aO\, kO \\ 0 & \frac{mO\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}{mN\, (\lambda - mO\, \Sigma O)} \end{pmatrix}$$

In[142]:= "The LN curve is downward sloping, whereas the LS curve is vertical";

In[143]:= **"1. A FALL IN aO, BETTER OUTSOURCING TECHNOLOGY";**

In[144]:= Clear [B, dr]

In[145]:= B = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {aO}],
gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {aO}]} // FullSimplify

Out[145]= $$\left\{\left\{kO \left(iO + \frac{iN\, \lambda\, (dr + iN + iO + \mu)\, \Sigma iO}{(iN + iO + \mu)\, (\lambda - mO\, \Sigma O)}\right)\right\}, \left\{\frac{kO\, \lambda\, (dr + iN + iO + \mu)\, (iO\, mO + \lambda\, \mu)\, \Sigma iO}{mN\, (iN + iO + \mu)\, (\lambda - mO\, \Sigma O)}\right\}\right\}$$

In[146]:= impactaO = -Inverse[J].B.{ΔaO} // FullSimplify;

In[147]:= "The expressions are too complicated and hence suppressed. I set";
dr = 0; impactaO = -Inverse[J].B.{ΔaO} // FullSimplify

Out[147]=
$$\left\{-\Delta aO\,(\lambda - mO\,\Sigma O)\left(\frac{\frac{iO}{\lambda\,\Sigma iO} + \frac{iN}{\lambda - mO\,\Sigma O}}{aO} - \frac{kO\,(iO\,mO + \lambda\,\mu)}{mO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}\right),\ -\frac{kO\,\Delta aO\,\lambda\,(iO\,mO + \lambda\,\mu)\,\Sigma iO}{mO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}\right\}$$

In[148]:= {diN, diO} = %;

In[149]:= "1.1. What happens to innovation rates: diN and diO?";

In[150]:= "The above implies that $\frac{diN}{daO}$ is ambiguous. To find an if and only if condition, I collect the iN and iO terms";

In[151]:= Collect[diN, {iO, iN}]

Out[151]=
$$-\frac{iN\,\Delta aO}{aO} + \frac{kO\,\Delta aO\,\lambda\,\mu\,(\lambda - mO\,\Sigma O)}{mO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))} - iO\,\Delta aO\,(\lambda - mO\,\Sigma O)\left(\frac{1}{aO\,\lambda\,\Sigma iO} - \frac{kO}{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}\right)$$

In[152]:= "--The first term in front of iN is negative.
--The second term with μ in it is positive.
--The third term in front of iO, which boils down to $\frac{aN\,iO\,kN\,\Delta aO\,(\lambda - mO\,\Sigma O)^2}{aO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}$, is positive.";

In[153]:= "Obviously there is an ambiguity. For diN<0, iN needs to be larger than a critical level. To find this I use"; Solve[diN == 0, iN] // FullSimplify

Out[153]=
$$\left\{\left\{iN \to \frac{(\lambda - mO\,\Sigma O)\,(aO\,kO\,\lambda^2\,\mu\,\Sigma iO + aN\,iO\,kN\,mO\,(\lambda - mO\,\Sigma O))}{mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}\right\}\right\}$$

In[154]:= "Thus we can state

$$\frac{diN}{daO}<0 \quad \text{iff} \quad iN > \frac{(\lambda - mO\,\Sigma O)\,(aO\,kO\,\lambda^2\,\mu\,\Sigma iO + aN\,iO\,kN\,mO\,(\lambda - mO\,\Sigma O))}{mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}$$";

In[155]:= "1.2.What happens to aggregate innovation: diA= diN+diO?";

In[156]:= diA = diN + diO // FullSimplify

Out[156]=
$$\left(\Delta aO\,\left(-aO\,kO\,\lambda^2\,\Sigma iO\,(\lambda\,\mu\,(-1 + \Sigma iO) + (iN + iO)\,mO\,\Sigma iO + mO\,\mu\,\Sigma O) + aN\,kN\,mO\,(\lambda - mO\,\Sigma O)\,(iN\,\lambda\,\Sigma iO + iO\,(\lambda - mO\,\Sigma O))\right)\right) \Big/ (aO\,mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)))$$

In[157]:= "Again, $\frac{diA}{daO}$ looks ambigous. To simplify, I collect the iN and iO terms";

In[158]:= **Collect[diA, {iO, iN}]**

Out[158]= $$\frac{\triangle aO\left(-aO\,kO\,\lambda^3\,\mu\,(-1+\Sigma iO)\,\Sigma iO - aO\,kO\,mO\,\lambda^2\,\mu\,\Sigma iO\,\Sigma O\right)}{aO\,mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))} + \frac{iN\,\triangle aO\left(-aO\,kO\,mO\,\lambda^2\,\Sigma iO^2 + aN\,kN\,mO\,\lambda\,\Sigma iO\,(\lambda - mO\,\Sigma O)\right)}{aO\,mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))} + \frac{iO\,\triangle aO\left(-aO\,kO\,mO\,\lambda^2\,\Sigma iO^2 + aN\,kN\,mO\,(\lambda - mO\,\Sigma O)^2\right)}{aO\,mO\,\lambda\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}$$

In[159]:= **"--The sign of first term after simplifying is the same as that of $\lambda(1-\Sigma iO)-mO\ \Sigma O$. For this to be negative, we need $\sigma iO < \frac{mo * \Sigma O}{\lambda}$.**

--The second term in front of iN is negative because of the resriction w>0.

--The third term in front of iO is negative. To see this note that the first term in the numerator which is negative is multiplied by $\lambda>1$, where as the second term which is positive is multiplied by $(\lambda-mO\ \Sigma O)$. Given $\lambda > \lambda$ - mo always holds, it follows that the entire term remains negative. The restriction w>0 is also used here.";

"Thus we conclude:

$\frac{diA}{daO}<0$ if $\sigma iO < \frac{mo * \Sigma O}{\lambda}$, a sufficient but hardly a necessary condition. ";

In[161]:= **"1.3.What happens to the fraction of industries?";**

In[162]:= **"Totally differentiating nO implies ";**
dnO = D[nO, iO] diO + D[nO, iN] diN // FullSimplify

Out[163]= $$-\left(\triangle aO\left(aO\,kO\,\lambda^2\,\mu\,\Sigma iO\,(\lambda\,(iN+\mu)\,\Sigma iO + iO\,(\lambda + mO\,(\Sigma iO - \Sigma O))) + aN\,iO\,kN\,mO\,(\lambda - mO\,\Sigma O)\,(iN\,\lambda\,\Sigma iO + iO\,(\lambda - mO\,\Sigma O))\right)\right) \Big/ \left(aO\,mO\,\lambda\,(iN + iO + \mu)^2\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))\right)$$

In[164]:= **"Clearly, $\frac{dnO}{daO}<0$**

a. If $(\Sigma iO-\Sigma O)>0$.

b. Otherwise, that is for $(\Sigma iO-\Sigma O)<0$, we need $(\Sigma O-\Sigma iO)< \frac{\lambda}{mO}$, which means that the absolute value of $(\Sigma iO-\Sigma O)$ needs to be sufficiently small

The part a of the condition basically implies $\sigma O > \sigma iO$, which is not a strict restriction";

In[165]:= **"Totally differentiating nN implies ";**
dnN = D[nN, iO] diO + D[nN, iN] diN // FullSimplify

Out[166]= $$\left(\triangle aO\left(aO\,kO\,\lambda^2\,\mu\,\Sigma iO\,(-iN\,mO\,\Sigma iO + \lambda\,(iO + \mu + iN\,\Sigma iO) - mO\,(iO+\mu)\,\Sigma O) + aN\,kN\,mO\,(iO+\mu)\,(\lambda - mO\,\Sigma O)\,(iN\,\lambda\,\Sigma iO + iO\,(\lambda - mO\,\Sigma O))\right)\right) \Big/ \left(aO\,mO\,\lambda\,(iN + iO + \mu)^2\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))\right)$$

In[167]:= **"Clearly, $\frac{dnN}{daO}$><0";**

In[168]:=
```
"Totally differentiating nS implies";
dnS = D[nS, iO] diO + D[nS, iN] diN // FullSimplify
```

Out[169]=
$$\left(\Delta aO\,\mu\left(aO\,kO\,\lambda^2\,\Sigma iO\,(\lambda\,\mu\,(-1+\Sigma iO)+(iN+iO)\,mO\,\Sigma iO+mO\,\mu\,\Sigma O)-aN\,kN\,mO\,(\lambda-mO\,\Sigma O)\,(iN\,\lambda\,\Sigma iO+iO\,(\lambda-mO\,\Sigma O))\right)\right)\Big/\left(aO\,mO\,\lambda\,(iN+iO+\mu)^2\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))\right)$$

In[170]:= "Clearly, $\frac{dnS}{daO}$><0. Note that nS moves inversely with iA. Thus:

$\frac{dnS}{daO}>0$ if $\sigma iO<\frac{mo*\Sigma O}{\lambda}$";

In[171]:= **"1.4. What happens to expenditure c?";**

In[172]:= "Let's totally differentiate";

In[173]:=
```
dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] ΔaO // FullSimplify
```

Out[173]=
$$-\frac{\Delta aO\left(aO\,kO\,\lambda^2\,\mu\,\Sigma iO\,(\lambda-\lambda\,\Sigma iO+mO\,(\Sigma iO-\Sigma O))+aN\,kN\,mO\,(\lambda-mO\,\Sigma O)\,(iN\,\lambda\,\Sigma iO+iO\,(\lambda-mO\,\Sigma O))\right)}{aO\,mN\,mO\,\Sigma iO\,(-\lambda+mO\,\Sigma O)}$$

In[174]:= "So, $\frac{dc}{daO}>0$ if $(\Sigma iO-\Sigma O)>0$ or small,
which alternatively can be stated as $\sigma O > \sigma iO$.";

In[175]:= **"2. A FALL IN ΣO, HIGHER MANUFACTURING SUBSIDY";**

In[176]:=
```
Clear [B, dr]
```

In[177]:=
```
B = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣO}],
  gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣO}]} // FullSimplify
```

Out[177]=
$$\left\{\left\{\frac{aO\,iN\,kO\,mO\,\lambda\,(dr+iN+iO+\mu)\,\Sigma iO}{(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)^2}\right\},\left\{\frac{aO\,kO\,mO\,\lambda\,(dr+iN+iO+\mu)\,(iO\,mO+\lambda\,\mu)\,\Sigma iO}{mN\,(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)^2}\right\}\right\}$$

In[178]:=
```
B = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣO}],
  gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣO}]}
```

Out[178]=
$$\left\{\left\{\frac{aO\,iN\,kO\,mO\,\lambda\,(dr+iN+iO+\mu)\,\Sigma iO}{(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)^2}\right\},\left\{\frac{aN\,kN\,mO\,(dr+iN+iO+\mu)\,(iO\,mO+\lambda\,\mu)}{mN\,(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)}+\frac{mO\,(dr+iN+iO+\mu)\,(iO\,mO+\lambda\,\mu)\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}{mN\,(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)^2}\right\}\right\}$$

In[179]:=
```
impactΣO = -Inverse[J].B.{ΔΣO} // FullSimplify;
```

In[180]:= "The expressions are too complicated so I set dr=0";

In[181]:= `dr = 0; impactΣO = -Inverse[J].B.{ΔΣO} // FullSimplify`

Out[181]= $$\left\{\frac{\text{iN mO }\Delta\Sigma O}{-\lambda + \text{mO }\Sigma O} + \frac{\text{aO kO }\Delta\Sigma O\ (\text{iO mO} + \lambda\,\mu)}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)},\ \frac{\text{aO kO }\Delta\Sigma O\ \lambda\ (\text{iO mO} + \lambda\,\mu)\ \Sigma\text{iO}}{(\lambda - \text{mO }\Sigma O)\ (-\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(\lambda - \text{mO }\Sigma O))}\right\}$$

In[182]:= `{diN, diO} = %;`

In[183]:= **"2.1. What happens to innovation rates: diN and diO?";**

In[184]:= "The above implies that $\frac{\text{diN}}{\text{d}\Sigma\text{O}}$ is ambiguous.
To find a necessary condition, I collect the iN and iO terms";

In[185]:= `Collect[diN, {iO, iN}]`

Out[185]= $$\frac{\text{iN mO }\Delta\Sigma O}{-\lambda + \text{mO }\Sigma O} + \frac{\text{aO iO kO mO }\Delta\Sigma O}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)} + \frac{\text{aO kO }\Delta\Sigma O\ \lambda\,\mu}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)}$$

In[186]:= "--The first expression is negative
--The second expression is positive (note that the denominator is positive for w >0).
--The third expression is positive.
Thus, $\frac{\text{diN}}{\text{d}\Sigma\text{O}}$ is ambiguous.";

In[187]:= "For a necessary condition, I note that iN must be a larger than a critical level";
`Solve[diN == 0, iN] // FullSimplify`

Out[187]= $$\left\{\left\{\text{iN} \to -\frac{\text{aO kO }(\text{iO mO} + \lambda\,\mu)\ (-\lambda + \text{mO }\Sigma O)}{\text{mO }(\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O))}\right\}\right\}$$

In[188]:= "Hence

$$\frac{\text{diN}}{\text{d}\Sigma\text{O}}<0 \quad \text{iff} \quad \text{iN}>\frac{\text{aO kO }(\text{iO mO} + \lambda\,\mu)\ (\lambda - \text{mO }\Sigma O)}{\text{mO }(\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O))}$$ ";

In[189]:= "On the other hand, $\frac{\text{diO}}{\text{d}\Sigma\text{O}}<0$ unambiguously ";

In[190]:= **"2.2.What happens to aggregate innovation: diA= diN+diO?";**

In[191]:= `diA = diN + diO // FullSimplify`

Out[191]= $$\frac{\Delta\Sigma O\ ((\text{iN} + \text{iO})\ \text{mO} + \lambda\,\mu)}{-\lambda + \text{mO }\Sigma O} - \frac{(\text{aN kN} - \text{aO kO})\ \Delta\Sigma O\ (\text{iO mO} + \lambda\,\mu)}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)}$$

In[192]:= "Again, $\frac{\text{diA}}{\text{d}\Sigma\text{O}}$ is ambigous. To simplify, I collect the iN and iO terms";

In[193]:= `Collect[diA, {iO, iN}]`

Out[193]= $$\frac{\text{iN mO }\Delta\Sigma O}{-\lambda + \text{mO }\Sigma O} + \frac{\Delta\Sigma O\ \lambda\,\mu}{-\lambda + \text{mO }\Sigma O} + \frac{(-\text{aN kN} + \text{aO kO})\ \Delta\Sigma O\ \lambda\,\mu}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)} +$$
$$\text{iO}\left(\frac{\text{mO }\Delta\Sigma O}{-\lambda + \text{mO }\Sigma O} + \frac{(-\text{aN kN} + \text{aO kO})\ \text{mO }\Delta\Sigma O}{\text{aO kO }\lambda\,\Sigma\text{iO} + \text{aN kN }(-\lambda + \text{mO }\Sigma O)}\right)$$

In[194]:= "--The first term in front of iN is negative
--The second term is negative.
--The third term is positive given aN kN-aO kO < 0. This is due to the restriction for w that mN w > mO*ΣO
--The fourth term in front of iO can be positive or negative. Note that first term is negative and the second term is positive. In the second term, the numerator is negative aN kN-aO kO <0 and the denominator is negative because of for w>0.";

"To simplify the fourth term, I cross-multiply terms and obtain the new numerator. This gives";
mO ΔΣO * (aO kO λ ΣiO + aN kN (-λ + mO ΣO)) + (-aN kN + aO kO) mO ΔΣO (-λ + mO ΣO) // FullSimplify

Out[196]= $aO\, kO\, mO\, \Delta\Sigma O\, (\lambda\, (-1 + \Sigma iO) + mO\, \Sigma O)$

In[197]:= "Note that the denominator is negative (one negative and one positive multiplied). The above will be positive iff σi0<mO*ΣO/λ, where σiO is the subsidy rate to outsourcing technology (recall that ΣiO=1-σiO. With ΣiO close to 1, that is, with σiO going to zero, we establish that the term in front of iO is negative.

Hence, the only term that is positive is associated with the imitation rate. We can identify this but we cannot sign diA unambiguously." ;

In[198]:= "To find a necessary condition"; Solve[diA == 0, iN] // FullSimplify

Out[198]= $\left\{\left\{iN \to -\frac{aO\, kO\, (iO\, mO + \lambda\, \mu)\, (\lambda\, (-1 + \Sigma iO) + mO\, \Sigma O)}{mO\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}\right\}\right\}$

In[199]:= " $\frac{diA}{d\Sigma O}<0$ iff $iN>-\frac{aO\, kO\, (iO\, mO + \lambda\, \mu)\, (\lambda\, (-1 + \Sigma iO) + mO\, \Sigma O)}{mO\, (aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O))}$ ";

In[200]:= "Hence if (λ (-1+ΣiO)+mO ΣO)>0, which holds when σi0<mO*ΣO/λ, the above will also automatically hold. Thus,

$\frac{diA}{d\Sigma O}<0$ if $\sigma i0<\frac{mO * \Sigma O}{\lambda}$ ";

In[201]:= "2.3.What happens to the fraction of industries?";

In[202]:= "Totally differentiating nO implies ";
dnO = D[nO, iO] diO + D[nO, iN] diN // FullSimplify

Out[203]= $\frac{\Delta\Sigma O\, \left(aN\, iN\, iO\, kN\, mO\, (\lambda - mO\, \Sigma O) + aO\, kO\, \left(\lambda^2\, \mu\, (iN + \mu)\, \Sigma iO + iO\, \lambda\, \mu\, (\lambda + mO\, (\Sigma iO - \Sigma O)) + iO^2\, mO\, (\lambda - mO\, \Sigma O)\right)\right)}{(iN + iO + \mu)^2\, (\lambda - mO\, \Sigma O)\, (-aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (\lambda - mO\, \Sigma O))}$

In[204]:= "Clearly, $\frac{dnO}{d\Sigma O}<0$ if (ΣiO-ΣO)>0 or small,

a. If (ΣiO-ΣO)>0.

b. Otherwise, that is for (ΣiO-ΣO)<0, we need (ΣO-ΣiO)< $\frac{\lambda}{mO}$, which means that the absolute value of (ΣiO-ΣO) needs to be sufficiently small

The part 'a' of the condition basically implies σO > σiO, which is not a strict restriction";

In[205]:=
```
"Totally differentiating nN implies ";
dnN = D[nN, iO] diO + D[nN, iN] diN // FullSimplify
```

Out[206]=

$$\big(\Delta\Sigma O\,(aN\,iN\,kN\,mO\,(iO+\mu)\,(-\lambda+mO\,\Sigma O)+aO\,kO\,(-\lambda\,(iO+\mu)\,(iO\,mO+\lambda\,\mu)+iN\,(mO-\lambda)\,\lambda\,\mu\,\Sigma iO+mO\,(iO+\mu)\,(iO\,mO+\lambda\,\mu)\,\Sigma O))\big)\Big/\Big((iN+iO+\mu)^2\,(\lambda-mO\,\Sigma O)\,(-aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(\lambda-mO\,\Sigma O))\Big)$$

In[207]:=
```
"Clearly, dnN/dΣO ><0";
```

In[208]:=
```
"Totally differentiating nS implies";
dnS = D[nS, iO] diO + D[nS, iN] diN // FullSimplify
```

Out[209]=

$$-\big(\Delta\Sigma O\,\mu\,(aN\,iN\,kN\,mO\,(-\lambda+mO\,\Sigma O)+aO\,kO\,(iO\,mO\,(\lambda\,(-1+\Sigma iO)+mO\,\Sigma O)+\lambda\,(\lambda\,\mu\,(-1+\Sigma iO)+mO\,(iN\,\Sigma iO+\mu\,\Sigma O))))\big)\Big/\Big((iN+iO+\mu)^2\,(\lambda-mO\,\Sigma O)\,(-aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(\lambda-mO\,\Sigma O))\Big)$$

In[210]:=
```
"Clearly, dnS/dΣO ><0. However, note that nS moves inversely with iA. Thus:
dnS/daO >0   if σi0< mO*ΣO/λ";
```

In[211]:=
```
"2.4. What happens to  expenditure c?";
```

In[212]:=
```
"Let's totally differentiate";
```

In[213]:=
```
dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] ΔΣO // FullSimplify
```

Out[213]=

$$\frac{1}{mN\,(\lambda-mO\,\Sigma O)^2}\Delta\Sigma O\,\lambda\,(aN\,iN\,kN\,mO\,(\lambda-mO\,\Sigma O)-kO\,(-\lambda\,(iN+iO+\mu)\,\Sigma iO\,(\lambda-mO\,\Sigma O)+aO\,(iO\,mO\,(\lambda\,(-1+\Sigma iO)+mO\,\Sigma O)+\lambda\,(\lambda\,\mu\,(-1+\Sigma iO)+mO\,(iN\,\Sigma iO+\mu\,\Sigma O)))))$$

In[214]:=
```
"So, dc/dΣO ><0";
```

In[215]:=
```
"3. A FALL IN ΣiO, HIGHER OUTSOURCING SUBSIDY";
```

In[216]:=
```
Clear [B, dr, ΣiO]
```

In[217]:=
```
B = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣiO}],
   gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {ΣiO}]} // FullSimplify
```

Out[217]=

$$\left\{\left\{\frac{aO\,iN\,kO\,\lambda\,(dr+iN+iO+\mu)}{(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)}\right\},\left\{\frac{aO\,kO\,\lambda\,(dr+iN+iO+\mu)\,(iO\,mO+\lambda\,\mu)}{mN\,(iN+iO+\mu)\,(\lambda-mO\,\Sigma O)}\right\}\right\}$$

In[218]:=
```
impactΣiO = -Inverse[J].B.{ΔΣiO} // FullSimplify;
```

In[219]:= **"The expressions are too complicated, thus, I set";**
dr = 0; impactΣiO = -Inverse[J].B.{ΔΣiO} // FullSimplify

Out[219]=
$$\left\{-\frac{\Delta\Sigma iO\left(iN+\frac{aO\,kO\,(iO\,mO+\lambda\,\mu)\,(-\lambda+mO\,\Sigma O)}{mO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}\right)}{\Sigma iO},\ -\frac{aO\,kO\,\Delta\Sigma iO\,\lambda\,(iO\,mO+\lambda\,\mu)}{mO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}\right\}$$

In[220]:= **{diN, diO} = %;**

In[221]:= **"3.1. What happens to innovation rates: diN and diO?";**

In[222]:= **"It looks like $\frac{diN}{daO}$ is ambiguous, so I collect the iN and iO terms";**

In[223]:= **Collect[diN, {iO, iN}]**

Out[223]=
$$-\frac{iN\,\Delta\Sigma iO}{\Sigma iO}-\frac{aO\,iO\,kO\,\Delta\Sigma iO\,(-\lambda+mO\,\Sigma O)}{\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}-\frac{aO\,kO\,\Delta\Sigma iO\,\lambda\,\mu\,(-\lambda+mO\,\Sigma O)}{mO\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}$$

In[224]:= **"--The first expression is negative.**
-The second expression is positive (note that the denominator is positive for w >0).
--The third expression is positive.

Hence $\frac{diN}{d\Sigma iO}$ is ambiguous";

In[225]:= **"I now look for a critical condition which will require that iN be above a critical level";**
Solve[diN == 0, iN] // FullSimplify

Out[225]=
$$\left\{\left\{iN\to-\frac{aO\,kO\,(iO\,mO+\lambda\,\mu)\,(-\lambda+mO\,\Sigma O)}{mO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}\right\}\right\}$$

In[226]:= **"Hence a cleaner expression is**

$\frac{diN}{d\Sigma iO}<0$ iff $iN>\frac{aO\,kO\,(iO\,mO+\lambda\,\mu)\,(\lambda-mO\,\Sigma O)}{mO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O))}$

Observe that this is the same condition that is required for $\frac{diN}{d\Sigma O}<0$";

In[227]:= **"On the other hand, $\frac{diO}{d\Sigma iO}<0$ unambiguously ";**

In[228]:= **"3.2.What happens to aggregate innovation: diA= diN+diO?";**

In[229]:= **diA = diN + diO // FullSimplify**

Out[229]=
$$-(\Delta\Sigma iO\,(aN\,iN\,kN\,mO\,(-\lambda+mO\,\Sigma O)+aO\,kO\,(iO\,mO\,(\lambda\,(-1+\Sigma iO)+mO\,\Sigma O)+\lambda\,(\lambda\,\mu\,(-1+\Sigma iO)+mO\,(iN\,\Sigma iO+\mu\,\Sigma O))))))\,/\,(mO\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO+aN\,kN\,(-\lambda+mO\,\Sigma O)))$$

In[230]:= **"Again, $\frac{diA}{d\Sigma iO}$ is ambigous. To simplify, I collect the iN and iO terms";**

In[231]:= **Collect[diA, {iO, iN}]**

Out[231]= $$-\frac{aO\ iO\ kO\ \Delta\Sigma iO\ (\lambda\ (-1+\Sigma iO)+mO\ \Sigma O)}{\Sigma iO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))}-\frac{\Delta\Sigma iO\ \left(aO\ kO\ \lambda^2\ \mu\ (-1+\Sigma iO)+aO\ kO\ mO\ \lambda\ \mu\ \Sigma O\right)}{mO\ \Sigma iO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))}-\frac{iN\ \Delta\Sigma iO\ (aO\ kO\ mO\ \lambda\ \Sigma iO+aN\ kN\ mO\ (-\lambda+mO\ \Sigma O))}{mO\ \Sigma iO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))}$$

In[232]:= **"The coefficient in front of iN is negative, so a necessary condition would require that iN be above a critical level"; Solve[diA == 0, iN] // FullSimplify**

Out[232]= $$\left\{\left\{iN\to-\frac{aO\ kO\ (iO\ mO+\lambda\ \mu)\ (\lambda\ (-1+\Sigma iO)+mO\ \Sigma O)}{mO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))}\right\}\right\}$$

In[233]:= **"** $\frac{diA}{d\Sigma iO}<0$ **iff** $iN>\frac{aO\ kO\ (iO\ mO+\lambda\ \mu)\ (\lambda\ (1-\Sigma iO)-mO\ \Sigma O)}{mO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))}$ **";**

In[234]:= **"Hence if (λ (1−ΣiO)−mO ΣO)<0, which holds when σi0<mO*ΣO/λ, then the above will automatically hold.Thus,**

$$\frac{diA}{d\Sigma iO}<0\quad \text{if}\ \sigma i0<\frac{mO*\Sigma O}{\lambda}.$$

Observe that this is the same condition as observed in the case of ΣO";

In[235]:= **"3.3.What happens to the fraction of industries?";**

In[236]:= **"Totally differentiating nO implies ";**
dnO = D[nO, iO] diO + D[nO, iN] diN // FullSimplify

Out[237]= $$\left(\Delta\Sigma iO\ \left(aN\ iN\ iO\ kN\ mO\ (-\lambda+mO\ \Sigma O)+aO\ kO\ \left(-\lambda^2\ \mu\ (iN+\mu)\ \Sigma iO-iO\ \lambda\ \mu\ (\lambda+mO\ (\Sigma iO-\Sigma O))+iO^2\ mO\ (-\lambda+mO\ \Sigma O)\right)\right)\right)\Big/\left(mO\ (iN+iO+\mu)^2\ \Sigma iO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))\right)$$

In[238]:= **"Clearly,** $\frac{dnO}{d\Sigma iO}<0$ **if (ΣiO−ΣO)>0 or small,**

which can also be stated as σO > σiO. Note that the denominator is negative";

In[239]:= **"Totally differentiating nN implies ";**
dnN = D[nN, iO] diO + D[nN, iN] diN // FullSimplify

Out[240]= $$(\Delta\Sigma iO\ (aN\ iN\ kN\ mO\ (iO+\mu)\ (\lambda-mO\ \Sigma O)+aO\ kO\ (\lambda\ (iO+\mu)\ (iO\ mO+\lambda\ \mu)+iN\ \lambda\ (-mO+\lambda)\ \mu\ \Sigma iO-mO\ (iO+\mu)\ (iO\ mO+\lambda\ \mu)\ \Sigma O)))\Big/\left(mO\ (iN+iO+\mu)^2\ \Sigma iO\ (aO\ kO\ \lambda\ \Sigma iO+aN\ kN\ (-\lambda+mO\ \Sigma O))\right)$$

In[241]:= **"Clearly,** $\frac{dnN}{\Sigma iO}$**><0";**

In[242]:= "Totally differentiating nS implies";
dnS = D[nS, iO] diO + D[nS, iN] diN // FullSimplify

Out[243]= $(\Delta\Sigma iO\,\mu\,(aN\,iN\,kN\,mO\,(-\lambda + mO\,\Sigma O) + aO\,kO\,(iO\,mO\,(\lambda\,(-1 + \Sigma iO) + mO\,\Sigma O) + \lambda\,(\lambda\,\mu\,(-1 + \Sigma iO) + mO\,(iN\,\Sigma iO + \mu\,\Sigma O)))))\,/\,\left(mO\,(iN + iO + \mu)^2\,\Sigma iO\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))\right)$

In[244]:= "Clearly, $\frac{dnS}{d\Sigma iO}$ looks ambiguous. However, note that nS moves inversely with iA. Thus:

$\frac{dnS}{d\Sigma iO}>0$ if $\sigma i0<\frac{mO * \Sigma O}{\lambda}$";

In[245]:= **"3.4. What happens to expenditure c?";**

In[246]:= "Let's totally differentiate";

In[247]:= dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] ΔΣOi // FullSimplify

Out[247]= $\frac{1}{mN\,mO\,\Sigma iO\,(-\lambda + mO\,\Sigma O)}\lambda\left(aN\,iN\,kN\,mO\,\Delta\Sigma iO\,(-\lambda + mO\,\Sigma O) + kO\left(-mO\,\Delta\Sigma Oi\,\lambda\,(iN + iO + \mu)\,\Sigma iO^2 + aO\,\Delta\Sigma iO\,(iO\,mO\,(\lambda\,(-1 + \Sigma iO) + mO\,\Sigma O) + \lambda\,(\lambda\,\mu\,(-1 + \Sigma iO) + mO\,(iN\,\Sigma iO + \mu\,\Sigma O)))\right)\right)$

In[248]:= "So, $\frac{dc}{d\Sigma iO}$><0";

In[249]:= **"4. A FALL IN μ, STRONGER IPRs";**

In[250]:= Clear [B, dr]

In[251]:= B = {gradf[F1[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {μ}],
gradf[F2[iN, iO, μ, aO, aN, mN, mO, ηS, λ, dr, kN, kO, ΣO, ΣiO], {μ}]} // FullSimplify

Out[251]= $\left\{\left\{\frac{dr\,iN\,(-aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(\lambda - mO\,\Sigma O))}{(iN + iO + \mu)^2\,(\lambda - mO\,\Sigma O)}\right\}, \left\{\frac{\left(-dr\,iO\,mO + dr\,(iN + iO)\,\lambda + \lambda\,(iN + iO + \mu)^2\right)\,(aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O))}{mN\,(iN + iO + \mu)^2\,(\lambda - mO\,\Sigma O)}\right\}\right\}$

In[252]:= impactμ = -Inverse[J].B.{Δμ} // FullSimplify;

In[253]:=
```
"The expressions are too complicated, thus, I set";
dr = 0; impactμ = -Inverse[J].B.{Δμ} // FullSimplify
```

Out[253]= $\left\{\frac{\Delta\mu\ (\lambda - \text{mO}\ \Sigma\text{O})}{\text{mO}\ \Sigma\text{iO}},\ -\frac{\Delta\mu\ \lambda}{\text{mO}}\right\}$

In[254]:=
```
{diN, diO} = %;
```

In[255]:=
```
"3.1. What happens to innovation rates: diN and diO?";
```

In[256]:= "Clearly, $\frac{\text{diN}}{\text{d}\mu}$>0 and $\frac{\text{diO}}{\text{d}\mu}$<0.";

In[257]:=
```
"3.2.What happens to aggregate innovation: diA= diN+diO?";
```

In[258]:=
```
diA = diN + diO // FullSimplify
```

Out[258]= $\frac{\Delta\mu\ (\lambda - \lambda\ \Sigma\text{iO} - \text{mO}\ \Sigma\text{O})}{\text{mO}\ \Sigma\text{iO}}$

In[259]:= "Note that (λ- λ ΣiO)-mO ΣO)<0 holds
when σi0<mO*ΣO/λ, which is a reasonable assumption. Thus,

$\frac{\text{diA}}{\text{d}\mu}$<0 if and only if σi0< $\frac{\text{mO} * \Sigma\text{O}}{\lambda}$ ";

In[260]:=
```
"3.3.What happens to the fraction of industries?";
```

In[261]:=
```
"Totally differentiating nO implies ";
dnO = D[nO, iO] diO + D[nO, iN] diN // FullSimplify
```

Out[262]= $\frac{-\Delta\mu\ \lambda\ (\text{iO} + (\text{iN} + \mu)\ \Sigma\text{iO}) + \text{iO}\ \text{mO}\ \Delta\mu\ \Sigma\text{O}}{\text{mO}\ (\text{iN} + \text{iO} + \mu)^2\ \Sigma\text{iO}}$

In[263]:= "Clearly, $\frac{\text{dnO}}{\text{d}\mu}$<0.";

In[264]:=
```
"Totally differentiating nN implies ";
dnN = D[nN, iO] diO + D[nN, iN] diN // FullSimplify
```

Out[265]= $\frac{\Delta\mu\ \lambda\ (\text{iO} + \mu + \text{iN}\ \Sigma\text{iO}) - \text{mO}\ \Delta\mu\ (\text{iO} + \mu)\ \Sigma\text{O}}{\text{mO}\ (\text{iN} + \text{iO} + \mu)^2\ \Sigma\text{iO}}$

In[266]:= "Clearly, $\frac{\text{dnN}}{\Sigma\mu}$><0";

In[267]:= "Totally differentiating nS implies";

```
dnS = D[nS, iO] diO + D[nS, iN] diN + D[nS, μ] Δμ // FullSimplify
```

Out[268]= $\frac{\Delta\mu\,(\lambda\,\mu\,(-1+\Sigma iO)+(iN+iO)\,mO\,\Sigma iO+mO\,\mu\,\Sigma O)}{mO\,(iN+iO+\mu)^2\,\Sigma iO}$

In[269]:= "Clearly, $\frac{dnS}{d\mu}>0$ if $\sigma i0<\frac{mO*\Sigma O}{\lambda}$";

In[270]:= "3.4. What happens to expenditure c?";

In[271]:= "Let's totally differentiate";

```
In[272]:= dc = D[c, iN] diN + D[c, iO] diO + D[c, aO] Δμ // FullSimplify
```

Out[272]= $\frac{1}{mN\,mO\,\Sigma iO\,(-\lambda+mO\,\Sigma O)}$
$\Delta\mu\,\lambda\,(-aN\,kN\,(\lambda-mO\,\Sigma O)\,(\lambda\,(-1+\Sigma iO)+mO\,\Sigma O)+kO\,\lambda\,\Sigma iO\,(-mO\,(iN+iO+\mu)\,\Sigma iO+aO\,(\lambda\,(-1+\Sigma iO)+mO\,\Sigma O)))$

In[273]:= "So, $\frac{dc}{d\mu}$><0";

```
In[274]:= "GLOBALIZATION, R&D and the iPod CYCLE";

In[275]:= "APPENDIX B:
NUMERICAL SIMULATIONS TO SIGN COMPARATIVE STATICS
THE MODEL WITH UNEQUAL IMITATION
   RATES TARGETING BOTH NORTHERN AND OUTSOURCING INDUSTRIES;
Date: August 24, 2007";

In[276]:= "CONSIDERATIONS IN BENCHMARK PARAMETER CHOICES:";

In[277]:= "1. Note that mO>1 must hold. Otherwise the current-quality leader has to
   compete the previous Northern outsourcer and not the Southern follower.
   This is not going to be compatible with the pricing scheme in the paper.
2. I impose aO*kO > aN*kN so that the value of an outsourcing
   firm is larger than that of a local-sourcing firm
3. Choose λ such that the mark up over marginal cost λ-mN*w is around
   25% in the North. Choosing a high λ also creates some room
   for the North-South relative wage to diverge significantly.
4. Choose aN and aO such that the aggregate rate of innovation
   is in the neighborhood of 2%.
5. Choose μ such that it is in the neighborhood of the aggregate innovation
   rate and it leads to a somehwat equal configuration of industries.
6. The mN and mO parameters below may look close to each other but
   actually the marginal costs in the North and the South differ
   substantially when the relative wage w is taken into account.
7. kN and kO are normalized to one because they always enter as multiplicative
   terms attached to aN and aO; hence variations in aN and aO are sufficient.
8. The terms REPO = iO+iN+μO and REPN = iO+iN+μN capture the threat
   of replacement faced by Outsourcing and Northern industries
9. Further details on the choice of parameters with specific references to empirical
   lierature can be found in my previous papers (such as Sener (2006), IPRs and Rent
   Protection in a North-South Product Cycle Model, manuscript, Sayek and Sener (2006),
   Outsourcing and Wage Inequality in a Dynamic Product Cycle Model, Appendix).";

In[278]:= Clear[FEIN, FEIO]; Clear[λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μN, μO];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];

In[279]:= "THE BENCHMARK PARAMETERS";

In[280]:= dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;

In[286]:= "STEADY-STATE EQUATIONS";

In[287]:= "I first note the equations for c and w in the imitation model. The long derivations
   are in the 'Unequal Imitation Steady-State Solutions Mathematica file";
```

In[288]:= "This is the equation for c";

In[289]:=
$$CON = c + \frac{\lambda\,(-aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(\lambda - mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)} == 0;$$

In[290]:= "This is the equation for North-South relative wage";

In[291]:=
$$WG = w - \frac{aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,(dr + iN + iO + \mu O)\,\Sigma iO} == 0;$$

In[292]:= "I enter the industry equilibrium shares";

In[293]:=
$$nO = \frac{iO}{iN + iO + \mu O};\ nN = \frac{iN}{iN + iO + \mu N};\ nS = \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu O};$$

In[294]:= "Profit flow of a Northern firm
adjusted for world population LN+LS, ie., divided by LN+LS ";

In[295]:=
$$\pi N = \frac{c\,(\lambda - mN * w)}{\lambda};$$

In[296]:= "Profit flow of an outsourcing firm adjusted for world population";

In[297]:=
$$\pi O = \frac{c\,(\lambda - (mO * (\Sigma O)))}{\lambda};$$

In[298]:= "I now rewrite the Northern and Southern labor market equations ";

In[299]:=
$$LN = (iN * aN * kN) + (iO * aO * kO) + \left(nN * \frac{c * mN}{\lambda}\right) - \frac{1}{(1 + \eta S)} == 0 \ // \ FullSimplify;$$

In[300]:=
$$LS = \frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)} == 0;$$

In[301]:= "NUMERICAL SOLUTIONS FOR STEADY-STATE EQUILIRBRIUM";

In[302]:=
```
Needs["Miscellaneous`RealOnly`"];NSolve[{LN,LS, CON, WG},{iN,iO,w,c}]
```

Out[302]=
```
{{iN → -0.606639, iO → 0.528835, w → 1.13544, c → -0.145671},
 {iN → -0.00140532, iO → -0.00421162, w → 1.2821, c → 0.934554},
 {iN → 0.005561, iO → 0.00724858, w → 1.27703, c → 1.21029}}
```

In[303]:=
```
FindRoot[{LN, LS, CON, WG}, {iN, 0.01}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]
```

Out[303]=
```
{iN → 0.005561, iO → 0.00724858, c → 1.21029, w → 1.27703}
```

In[304]:=
```
Output[1] = %;
```

In[305]:=
```
{"iA" → iN + iO, "nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
  "w*mN" → w * mN, "mO*ΣO" → mO * ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%
```

Out[305]=
```
{iA → 0.0128096, nN → 0.351749, nO → 0.30444, nS → 0.343812, πN → 0.128384,
 πO → 0.387294, w*mN → 1.34088, mO*ΣO → 1.02, REPO → 0.0238096, REPN → 0.0158096}
```

In[306]:=
```
"The rate of growth in utility is"; 0.012809578328133838` * Log[1.5]
```

Out[306]=
```
0.00519384
```

```
In[307]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * ΣO, πN, πO} /. %%%

Out[307]= {0.005561, 0.00724858, 0.0128096, 1.27703, 1.21029,
 0.351749, 0.30444, 0.343812, 1.34088, 1.02, 0.128384, 0.387294}

In[308]:= "OBSERVATIONS FOR NUMERICAL OUTCOMES";

In[309]:= "1. iA is around 1.2%, which renders g=iA*Log[1.5]=0.005.
   This is the growth rate attributable to technology improvements
   according to Denison (1985) and also used in Segerstrom papers.
2. The mark-up λ-mN*w is around 25%.
3. The industry configuration looks somewhat equally distributed and reasonable.
4. North-South wage gap is around 25%, which sounds
   low but indeed reasonable given the upper bound dictated by λ.
5. Profit flow per world population are positive and thus the model is correctly solved. ";

In[310]:= 0.005 / Log[1.5]

Out[310]= 0.0123315

In[311]:= "1. A DECLINE IN aO";

In[312]:= Clear [FEIN, FEIO]; Clear [λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μ];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];

In[313]:= dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;

In[319]:= aO = 3.8 * 0.9;
```

In[320]:=

$$CON = c + \frac{\lambda\,(-aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(\lambda - mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)} == 0;$$

$$WG = w - \frac{aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,(dr + iN + iO + \mu O)\,\Sigma iO} == 0;\ nO = \frac{iO}{iN + iO + \mu O};$$

$$nN = \frac{iN}{iN + iO + \mu N};\ nS = \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu O};\ \pi N = \frac{c\,(\lambda - mN * w)}{\lambda};\ \pi O = \frac{c\,(\lambda - (mO * (\Sigma O)))}{\lambda};$$

$$LN = (iN * aN * kN) + (iO * aO * kO) + \left(nN * \frac{c * mN}{\lambda}\right) - \frac{1}{(1 + \eta S)} == 0\ //\ FullSimplify;$$

$$LS = \frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)} == 0;$$

```
In[321]:= Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG}, {iN, iO, w, c}]

Out[322]= {{iN → -0.69836, iO → 0.620821, w → 1.09939, c → -0.123839},
 {iN → -0.00295836, iO → -0.00474188, w → 1.26668, c → 0.802781},
 {iN → 0.00846547, iO → 0.0155753, w → 1.25788, c → 1.22392}}
```

```
In[323]:= FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]

Out[323]= {iN → 0.00846547, iO → 0.0155753, c → 1.22392, w → 1.25788}

In[324]:= dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
          diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
          diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
          dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);

In[325]:= {"nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
            "w*mN" → w * mN, "mO*ΣO" → mO * ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%

Out[325]= {nN → 0.313063, nO → 0.444491, nS → 0.242446, πN → 0.146238,
           πO → 0.391653, w*mN → 1.32077, mO*ΣO → 1.02, REPO → 0.0350408, REPN → 0.0270408}

In[326]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * ΣO, πN, πO} /. %%%

Out[326]= {0.00846547, 0.0155753, 0.0240408, 1.25788, 1.22392,
           0.313063, 0.444491, 0.242446, 1.32077, 1.02, 0.146238, 0.391653}

In[327]:= {{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
           {"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}

Out[327]= {{dw → -0.0191517}, {diO → 0.00832672}, {diN → 0.00290447}, {diA → 0.0112312},
           {dnO → 0.140051}, {dc → 0.0136226}, {dnN → -0.0386854}, {dnS → -0.101366}}

In[328]:= "Expected outcome based on no imitation case:
          dw<0,    diO>0,    diN>O iff iO/iN <(1-σIO)ϕN(1+ϕO),    diA>0,    dnO>0,    dnN < 0";
```

In[329]:=
```
"Check"; {iO/iN < (λ ΣiO (aO kO λ ΣiO - aN kN (λ - mO ΣO)))/(aN kN (λ - mO ΣO)^2) /. Output[1],
 {iO/iN, (λ ΣiO (aO kO λ ΣiO - aN kN (λ - mO ΣO)))/(aN kN (λ - mO ΣO)^2)} /. Output[1]}
```
Out[329]= {True, {1.30347, 20.731}}

In[330]:=
```
"BOTTOM LINE";
```

In[331]:=
```
"The expected results based on the zero imitation model continue to hold. The analytical
   condition I identified for an increase in iN also holds under this simulation";
```

In[332]:=
```
"2. A DECLINE IN ΣO (HIGHER SUBSIDY TO MANUFACTURING)";
```

In[333]:=
```
Clear [FEIN, FEIO]; Clear [λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μ];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];
```

In[334]:=
```
dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;
```

In[340]:=
```
ΣO = 1 * 0.9;
```

In[341]:=
```
CON = c + (λ (-aO kO λ (dr + iN + iO + μO) ΣiO + aN kN (dr + iN + iO + μN) (λ - mO ΣO)))/(mN (λ - mO ΣO)) == 0;
WG = w - (aO kO λ (dr + iN + iO + μO) ΣiO + aN kN (dr + iN + iO + μN) (-λ + mO ΣO))/(aO kO mN (dr + iN + iO + μO) ΣiO) == 0; nO = iO/(iN + iO + μO);
nN = iN/(iN + iO + μN); nS = (iO + μN)/(iN + iO + μN) - iO/(iN + iO + μO); πN = (c (λ - mN * w))/λ; πO = (c (λ - (mO * (ΣO))))/λ;
LN = (iN * aN * kN) + (iO * aO * kO) + (nN * (c * mN)/λ) - 1/(1 + ηS) == 0 // FullSimplify;
LS = ((nO) * c * mO)/λ + (nS) * c - ηS/(1 + ηS) == 0;
```

In[342]:=
```
Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG}, {iN, iO, w, c}]
```
Out[343]= {{iN → -0.612184, iO → 0.53129, w → 1.10677, c → -0.150598},
 {iN → -0.00405946, iO → -0.00463834, w → 1.25238, c → 0.715111},
 {iN → 0.00977062, iO → 0.0220986, w → 1.24088, c → 1.20156}}

In[344]:=
```
FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]
```
Out[344]= {iN → 0.00977062, iO → 0.0220986, c → 1.20156, w → 1.24088}

In[345]:=
```
dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);
```

In[346]:=
```
{"nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
  "w*mN" → w * mN, "mO*ΣO" → mO * ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%
```

Out[346]=
```
{nN → 0.280208, nO → 0.515488, nS → 0.204304, πN → 0.157861,
 πO → 0.466204, w*mN → 1.30293, mO*ΣO → 0.918, REPO → 0.0428692, REPN → 0.0348692}
```

In[347]:=
```
y = {iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * ΣO, πN, πO} /. %%%
```

Out[347]=
```
{0.00977062, 0.0220986, 0.0318692, 1.24088, 1.20156,
 0.280208, 0.515488, 0.204304, 1.30293, 0.918, 0.157861, 0.466204}
```

In[348]:=
```
{{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
 {"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}
```

Out[348]=
```
{{dw → -0.0361481}, {diO → 0.01485}, {diN → 0.00420962}, {diA → 0.0190596},
 {dnO → 0.211049}, {dc → -0.00873678}, {dnN → -0.0715408}, {dnS → -0.139508}}
```

In[349]:=

"Expected outcome based on no imitation case:
dw<0,   diO>0,   diN>O iff

$$\frac{iO}{iN} < \frac{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,(\lambda - mO\,\Sigma O)}, \quad diA>0, \quad dnO>0, \quad dnN < 0";$$

In[350]:=

```
"Check"; {iO/iN < (aO kO λ ΣiO + aN kN (-λ + mO ΣO))/(aO kO (λ - mO ΣO)) /. Output[1],
  {iO/iN, (aO kO λ ΣiO + aN kN (-λ + mO ΣO))/(aO kO (λ - mO ΣO))} /. Output[1]}
```

Out[350]= {True, {1.30347, 2.2089}}

In[351]:= "The if and only if condition holds, the results are in the expected direction";

In[352]:= "BOTTOM LINE";

In[353]:= "The expected results based on the equal imitation model continue to hold. The analytical condition I identified for an increase in iN also holds under this simulation";

In[354]:= "3. A DECLINE IN ΣiO (HIGHER SUBSIDY TO OUTSOURCING TECHNOLOGY)";

In[355]:=

```
Clear [FEIN, FEIO]; Clear [λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μ];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];
```

In[356]:= "THE BENCHMARK PARAMETERS";

In[357]:=

```
dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;
```

In[363]:= ΣiO = 1 * 0.9;

In[364]:=

```
CON = c + (λ (-aO kO λ (dr + iN + iO + μO) ΣiO + aN kN (dr + iN + iO + μN) (λ - mO ΣO)))/(mN (λ - mO ΣO)) == 0;
WG = w - (aO kO λ (dr + iN + iO + μO) ΣiO + aN kN (dr + iN + iO + μN) (-λ + mO ΣO))/(aO kO mN (dr + iN + iO + μO) ΣiO) == 0; nO = iO/(iN + iO + μO);
nN = iN/(iN + iO + μN); nS = (iO + μN)/(iN + iO + μN) - iO/(iN + iO + μO); πN = (c (λ - mN * w))/λ; πO = (c (λ - (mO * (ΣO))))/λ;
LN = (iN * aN * kN) + (iO * aO * kO) + (nN * (c * mN)/λ) - 1/(1 + ηS) == 0 // FullSimplify;
LS = ((nO) * c * mO)/λ + (nS) * c - ηS/(1 + ηS) == 0;
```

In[365]:=

```
Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG}, {iN, iO, w, c}]
```

Out[366]=

```
{{iN → -0.60945, iO → 0.530076, w → 1.12046, c → -0.148183},
 {iN → -0.00299573, iO → -0.0047368, w → 1.2667, c → 0.802352},
 {iN → 0.00819856, iO → 0.0152389, w → 1.25799, c → 1.21591}}
```

```
In[367]:= FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]

Out[367]= {iN → 0.00819856, iO → 0.0152389, c → 1.21591, w → 1.25799}

In[368]:= dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
          diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
          diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
          dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);

In[369]:= {"nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
            "w*mN" → w * mN, "mO*ΣO" → mO * ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%

Out[369]= {nN → 0.310111, nO → 0.44251, nS → 0.247379, πN → 0.145188,
           πO → 0.389092, w*mN → 1.32089, mO*ΣO → 1.02, REPO → 0.0344375, REPN → 0.0264375}

In[370]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * ΣO, πN, πO} /. %%%

Out[370]= {0.00819856, 0.0152389, 0.0234375, 1.25799, 1.21591,
           0.310111, 0.44251, 0.247379, 1.32089, 1.02, 0.145188, 0.389092}

In[371]:= {{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
            {"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}

Out[371]= {{dw → -0.019042}, {diO → 0.00799032}, {diN → 0.00263756}, {diA → 0.0106279},
           {dnO → 0.13807}, {dc → 0.00561796}, {dnN → -0.0416373}, {dnS → -0.0964325}}

In[372]:= "Expected outcome based on no imitation case:
          dw<0,    diO>0,   diN>O iff
```

$$\frac{iO}{iN} < \frac{aO\, kO\, \lambda\, \Sigma iO + aN\, kN\, (-\lambda + mO\, \Sigma O)}{aO\, kO\, (\lambda - mO\, \Sigma O)}, \quad diA>0, \quad dnO>0, \quad dnN < 0";$$

In[373]:= "Check"; $\left\{\frac{iO}{iN} < \frac{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,(\lambda - mO\,\Sigma O)}\right.$ /. Output[1],
$\left\{\frac{iO}{iN}, \frac{aO\,kO\,\lambda\,\Sigma iO + aN\,kN\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,(\lambda - mO\,\Sigma O)}\right\}$ /. Output[1]}

Out[373]= {True, {1.30347, 2.44408}}

In[374]:= "BOTTOM LINE";

In[375]:= "The expected results based on the equal imitation model continue to hold. The analytical condition I identified for an increase in iN also holds under this simulation";

In[376]:= "4. A DECREASE IN μO and μN (LOWER IMITATION RATES DUE TO TRIPS)";

In[377]:= Clear [FEIN, FEIO]; Clear [λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μ];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];

In[378]:= "THE BENCHMARK PARAMETERS";

In[379]:= dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;

In[385]:= μN = 0.003 * 0.9; μO = 0.011 * 0.9;

In[386]:= CON = $c + \frac{\lambda\,(-aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(\lambda - mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)}$ == 0;

WG = $w - \frac{aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,(dr + iN + iO + \mu O)\,\Sigma iO}$ == 0; nO = $\frac{iO}{iN + iO + \mu O}$;

nN = $\frac{iN}{iN + iO + \mu N}$; nS = $\frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu O}$; πN = $\frac{c\,(\lambda - mN * w)}{\lambda}$; πO = $\frac{c\,(\lambda - (mO * (\Sigma O)))}{\lambda}$;

LN = $(iN * aN * kN) + (iO * aO * kO) + \left(nN * \frac{c * mN}{\lambda}\right) - \frac{1}{(1 + \eta S)}$ == 0 // FullSimplify;

LS = $\frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)}$ == 0;

In[387]:= Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG}, {iN, iO, w, c}]

Out[388]= {{iN → -0.602335, iO → 0.525866, w → 1.14542, c → -0.143763},
{iN → -0.00137533, iO → -0.00386175, w → 1.28014, c → 0.922177},
{iN → 0.00597715, iO → 0.00890156, w → 1.27516, c → 1.2232}}

```
In[389]:= FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]

Out[389]= {iN → 0.00597715, iO → 0.00890156, c → 1.2232, w → 1.27516}

In[390]:= dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
          diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
          diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
          dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);

In[391]:= {"nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
            "w∗mN" → w ∗ mN, "mO∗ΣO" → mO ∗ ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%

Out[391]= {nN → 0.340022, nO → 0.359242, nS → 0.300736, πN → 0.131355,
           πO → 0.391423, w∗mN → 1.33892, mO∗ΣO → 1.02, REPO → 0.0247787, REPN → 0.0175787}

In[392]:= y = {iN, iO, iN + iO, w, c, nN, nO, nS, w ∗ mN, mO ∗ ΣO, πN, πO} /. %%%

Out[392]= {0.00597715, 0.00890156, 0.0148787, 1.27516, 1.2232,
           0.340022, 0.359242, 0.300736, 1.33892, 1.02, 0.131355, 0.391423}
```

In[393]:=
```
{{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
 {"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}
```

Out[393]=
```
{{dw → -0.00187052}, {diO → 0.00165298}, {diN → 0.000416149}, {diA → 0.00206913},
 {dnO → 0.0400563}, {dc → 0.0129023}, {dnN → -0.0185305}, {dnS → -0.0215258}}
```

In[394]:=
```
"BOTTOM LINE";
```

In[395]:=
```
"A lower imitation rate for both μO and μN leads
   to higher aggregate innovation. Both local-sourcing directed R&D
   and outsourcing-directed R&D intensities attain higher levels.

Recall that the equal imitation rate model predicts that iN declines and iO increases. The
   above result is due to our considering a more reasonable case with μO>μN. In
   the other Mathematica file, we formally show that the impact of lower imitation
   rates on iN turns from negative to positive as the μO-μN>0 gap increases.

Note that the outsourcing-directed R&D intensity, iO,
   responds more. Thus the equilibrium share of outsourcing industries
   nO increases at the expense of Southern and Northern industries.

The North-South wage gap declines, because the profitability of
   local-sourcing directed R&D relative to outsourcing directed R&D goes down
   once all the change in innovation and imitation are taken into account.

Note that REPN and REPO both increase despite the fal in imitation rates. This is becuase
   iA increases and hence the pace of creative destruction. We can conclude though
   that the increase in the replacement rate as faced by the Northern producers
   (the increase in iA+μN) is proportionally larger than the increase in the
   replacement rate faced by the Outsourcing producers (the increase in iA+μO).";
```

In[396]:=
```
"5. A DECREASE IN μO (LOWER IMITATION TARGETED AT OUTSOURCING INDUSTRIES)";
```

In[397]:=
```
Clear[FEIN, FEIO]; Clear[λ, aN, aO, kN, kO, λ, ΣiO, ΣO, mN, mO, ηS, dr, μ];
Clear[LN, LS]; Clear[c, w, iN, iO, nN, nO, nS];
```

In[398]:=
```
"THE BENCHMARK PARAMETERS";
```

In[399]:=
```
dr = 0.07 - 0.014; μN = 0.003; μO = 0.011; λ = 1.5;
ηS = 2;
kN = 1; kO = 1;
aN = 1.4; aO = 3.8;
ΣO = 1; ΣiO = 1;
mN = 1.05; mO = 1.02;
```

In[405]:=
```
μO = 0.011 * 0.9;
```

In[406]:=
$$CON = c + \frac{\lambda\,(-aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(\lambda - mO\,\Sigma O))}{mN\,(\lambda - mO\,\Sigma O)} == 0;$$

$$WG = w - \frac{aO\,kO\,\lambda\,(dr + iN + iO + \mu O)\,\Sigma iO + aN\,kN\,(dr + iN + iO + \mu N)\,(-\lambda + mO\,\Sigma O)}{aO\,kO\,mN\,(dr + iN + iO + \mu O)\,\Sigma iO} == 0;\ nO = \frac{iO}{iN + iO + \mu O};$$

$$nN = \frac{iN}{iN + iO + \mu N};\ nS = \frac{iO + \mu N}{iN + iO + \mu N} - \frac{iO}{iN + iO + \mu O};\ \pi N = \frac{c\,(\lambda - mN * w)}{\lambda};\ \pi O = \frac{c\,(\lambda - (mO * (\Sigma O)))}{\lambda};$$

$$LN = (iN * aN * kN) + (iO * aO * kO) + \left(nN * \frac{c * mN}{\lambda}\right) - \frac{1}{(1 + \eta S)} == 0 \text{ // FullSimplify};$$

$$LS = \frac{(nO) * c * mO}{\lambda} + (nS) * c - \frac{\eta S}{(1 + \eta S)} == 0;$$

In[407]:=
```
Needs["Miscellaneous`RealOnly`"];
NSolve[{LN, LS, CON, WG}, {iN, iO, w, c}]
```

Out[408]=
```
{{iN → -0.601168, iO → 0.524801, w → 1.14913, c → -0.142836},
 {iN → -0.00112621, iO → -0.00395518, w → 1.27926, c → 0.923907},
 {iN → 0.00607262, iO → 0.00872964, w → 1.27455, c → 1.22145}}
```

In[409]:=
```
FindRoot[{LN, LS, CON, WG}, {iN, 0.02}, {iO, 0.01}, {c, 1}, {w, 1}, MaxIterations → 10 000]
```

Out[409]=
```
{iN → 0.00607262, iO → 0.00872964, c → 1.22145, w → 1.27455}
```

In[410]:=
```
dw = (w /. %) - (w /. Output[1]); dc = (c /. %) - (c /. Output[1]);
diN = (iN /. %) - (iN /. Output[1]); diO = (iO /. %) - (iO /. Output[1]);
diA = ((iO + iN) /. %) - ((iO + iN) /. Output[1]); dnN = (nN /. %) - (nN /. Output[1]);
dnO = (nO /. %) - (nO /. Output[1]); dnS = (nS /. %) - (nS /. Output[1]);
```

In[411]:=
```
{"nN" → nN, "nO" → nO, "nS" → nS, "πN" → πN, "πO" → πO,
  "w*mN" → w * mN, "mO*ΣO" → mO * ΣO, "REPO" → iO + iN + μO, "REPN" → iO + iN + μN} /. %%
```

Out[411]=
```
{nN → 0.341115, nO → 0.353394, nS → 0.30549, πN → 0.131691,
 πO → 0.390865, w*mN → 1.33828, mO*ΣO → 1.02, REPO → 0.0247023, REPN → 0.0178023}
```

In[412]:=
```
y = {iN, iO, iN + iO, w, c, nN, nO, nS, w * mN, mO * ΣO, πN, πO} /. %%%
```

Out[412]=
```
{0.00607262, 0.00872964, 0.0148023, 1.27455, 1.22145,
 0.341115, 0.353394, 0.30549, 1.33828, 1.02, 0.131691, 0.390865}
```

In[413]:=
```
{{"dw" → dw}, {"diO" → diO}, {"diN" → diN}, {"diA" → diA},
 {"dnO" → dnO}, {"dc" → dc}, {"dnN" → dnN}, {"dnS" → dnS}}
```

Out[413]=
```
{{dw → -0.00248238}, {diO → 0.00148106}, {diN → 0.000511625}, {diA → 0.00199269},
 {dnO → 0.0342084}, {dc → 0.0111584}, {dnN → -0.0106335}, {dnS → -0.0235749}}
```

In[414]:=
```
"BOTTOM LINE";
```

In[415]:=
```
"A lower imitation rate targeting the Outsourcing industries
   μO leads to higher aggregate innovation. Both local-sourcing directed
   R&D and outsourcing-directed R&D intensities attain higher levels.

Note that the outsourcing-directed R&D intensity, iO,
   responds more. Thus the equilibrium share of outsourcing industries
   nO increases at the expense of Southern and Northern industries.

The North-South wage gap declines, because the profitability of
   local-sourcing directed R&D relative to outsourcing directed R&D goes down
   once all the change in innovation and imitation are taken into account.

Note again that REPN and REPO increase. However, the increase in
   the replacement rate as faced by the Northern producers (the increase
   in iA+μN) is proportionally larger than the increase in the replacement
   rate faced by the Outsourcing producers (the increase in iA+μO).";
```